SAMPLE CALCULUS SCREENING TEST: EXPLANATIONS

These explanations are provided to help you understand the sample Screening Test questions. They are not the only possible explanations, although each question has only one correct answer.

You should try to solve the problems on your own before reading these solutions, even if it takes several attempts. Remember, there are 50 questions on the actual Screening Test, and these are just samples; do not expect that studying these explanations is all that you need to do to prepare for the test.

1. $\sqrt{2}+\sqrt{3}=$

(a) $\sqrt{5}$
(b) $\sqrt{6}$
(c) $2\sqrt{3}$
(d) 1
(e) none of the above

Answer: The correct answer is (e). To verify this it is necessary to eliminate the first four choices.

Method 1: These are just numbers, so we do some simple calculations (this does not require a calculator).

First, $(1.4)^2 = 1.96$ is smaller than 2 and $(1.5)^2 = 2.25$ is greater than than 2, so $1.4 < \sqrt{2} < 1.5$. Similarly, $(1.7)^2 = 2.89$ is smaller than 3 and $(1.8)^2 = 3.24$ is greater than 3, so $1.7 < \sqrt{3} < 1.8$. Since $1.4 + 1.7 = 3.1$ and $1.5 + 1.8 = 3.3$, we can add these inequalities to get

$$3.1 < \sqrt{2}+\sqrt{3} < 3.3$$

Now we can use this to eliminate the first four choices:

(a): Since $(2.3)^2 = 5.29 > 5$ we have $\sqrt{5} < 2.3$, so $\sqrt{5}$ is not between 3.1 and 3.3. So (a) is incorrect.

(b): Since $(2.5)^2 = 6.25 > 6$ we have $\sqrt{6} < 2.5$, so $\sqrt{6}$ is not between 3.1 and 3.3. So (b) is incorrect.

(c): Since $\sqrt{3} > 1.7$ (as calculated above) we have $2\sqrt{3} > 3.4$, so $2\sqrt{3}$ is not between 3.1 and 3.3. So (c) is incorrect.

(d): 1 is not between 3.1 and 3.3. So (d) is incorrect.

Method 2: We eliminate each of the the first four choices by assuming it is correct and finding a contradiction:

(a) is incorrect: Assume it is true and simplify:

$$\begin{aligned}
\sqrt{2}+\sqrt{3} &= \sqrt{5} \\
(\sqrt{2}+\sqrt{3})^2 &= (\sqrt{5})^2 \\
(\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 &= 5 \\
2 + 2\sqrt{2}\sqrt{3} + 3 &= 5 \\
5 + 2\sqrt{2}\sqrt{3} &= 5 \\
2\sqrt{2}\sqrt{3} &= 0
\end{aligned}$$

and the last line is obviously false since 2, $\sqrt{2}$ and $\sqrt{3}$ are all non-zero.

(b) is incorrect: Assume it is true and simplify:

$$\begin{aligned}
\sqrt{2}+\sqrt{3} &= \sqrt{6} \\
(\sqrt{2}+\sqrt{3})^2 &= (\sqrt{6})^2 \\
(\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 &= 6 \\
2 + 2\sqrt{2}\sqrt{3} + 3 &= 6 \\
5 + 2\sqrt{2}\sqrt{3} &= 6 \\
2\sqrt{2}\sqrt{3} &= 1 \\
(2\sqrt{2}\sqrt{3})^2 &= 1^2 \\
2^2 \cdot (\sqrt{2})^2(\sqrt{3})^3 &= 1 \\
4 \cdot 2 \cdot 3 &= 1 \\
24 &= 1
\end{aligned}$$

and the last line is obviously false.

(c) is incorrect: Assume it is true and simplify:

$$\begin{aligned}
\sqrt{2}+\sqrt{3} &= 2\sqrt{3} \\
(\sqrt{2}+\sqrt{3})^2 &= (2\sqrt{3})^2 \\
(\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 &= 2^2 \cdot (\sqrt{3})^2 \\
2 + 2\sqrt{2}\sqrt{3} + 3 &= 4 \cdot 3 \\
5 + 2\sqrt{2}\sqrt{3} &= 12 \\
2\sqrt{2}\sqrt{3} &= 7 \\
(2\sqrt{2}\sqrt{3})^2 &= 7^2 \\
4 \cdot (\sqrt{2})^2(\sqrt{3})^3 &= 49 \\
4 \cdot 2 \cdot 3 &= 49 \\
24 &= 49
\end{aligned}$$

and the last line is obviously false.

(d) is incorrect: Assume it is true and simplify:

$$\begin{aligned}
\sqrt{2}+\sqrt{3} &= 1 \\
(\sqrt{2}+\sqrt{3})^2 &= 1^2 \\
(\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 &= 1 \\
2 + 2\sqrt{2}\sqrt{3} + 3 &= 1 \\
5 + 2\sqrt{2}\sqrt{3} &= 1 \\
2\sqrt{2}\sqrt{3} &= -4
\end{aligned}$$

and the last line is obviously false, since 2, $\sqrt{2}$ and $\sqrt{3}$ are all positive.

2. $\dfrac{6^{20}}{3^{18}\cdot 2^{21}} =$

(a) $\dfrac{1}{6^{19}}$

(b) $\frac{20}{39}$

(c) $\frac{9}{2}$

(d) $\frac{1}{19}$

(e) none of the above

Answer: Replace 6 with $2\cdot 3$, so $6^{20} = (2\cdot 3)^{20} = 2^{20}\cdot 3^{20}$. Now just collect the powers of 2 and 3 and subtract exponents:

$$\frac{6^{20}}{3^{18}\cdot 2^{21}} = \frac{2^{20}\cdot 3^{20}}{3^{18}\cdot 2^{21}} = \frac{2^{20}}{2^{21}}\cdot\frac{3^{20}}{3^{18}} = 2^{20-21}\cdot 3^{20-18} = 2^{-1}\cdot 3^2 = \frac{1}{2}\cdot 9 = \frac{9}{2}$$

so the correct answer is (c).

3. $\left(x^{2/3}\right)^2 =$

(a) $\sqrt{x^{2/3}}$

(b) $x^{4/3}$

(c) $x^{2+2/3}$

(d) $x^{4/6}$

(e) none of the above

Answer: For this just use the law of exponents that says to multiply exponents when taking powers of powers: $(a^b)^c = a^{bc}$:

$$\left(x^{2/3}\right)^2 = x^{2\cdot(2/3)} = x^{4/3}$$

so the correct answer is (b).

4. $\dfrac{9x+6y}{3x} =$

(a) $3+2y$

(b) $\dfrac{3x+2y}{x}$

(c) $3x+2y$

(d) $3+6y$

(e) none of the above

Answer: You can factor 3 from the numerator and denominator and cancel it, so

$$\frac{9x+6y}{3x} = \frac{3(3x+2y)}{3x} = \frac{3x+2y}{x}$$

so the correct answer is (b).

5. Solve for x: $x^2 - 1 = 15$

(a) $x = \pm\sqrt{14}$
(b) $x = \pm 4$
(c) $x = \sqrt{15} + 1$
(d) $x = 4$
(e) none of the above

Answer: **Method 1:** Simplify and then take square roots:

$$\begin{aligned} x^2 - 1 &= 15 \\ x^2 &= 16 \\ x &= \pm\sqrt{16} \\ x &= \pm 4 \end{aligned}$$

so the correct answer is (b).

Method 2: Collect terms on one side and factor:

$$\begin{aligned} x^2 - 1 &= 15 \\ x^2 - 16 &= 0 \\ (x-4)(x+4) &= 0 \\ x &= 4 \text{ or } x = -4 \\ x &= \pm 4 \end{aligned}$$

so the correct answer is (b).

6. Solve for x: $|x| < 3$

(a) $x < 3$
(b) $x < \pm 3$
(c) $x > 3$ *or* $x < -3$
(d) $-3 < x < 3$
(e) none of the above

Answer: **Method 1:** $|x| < 3$ means that the value of x, ignoring the sign, is less than 3. This means that x cannot be larger than, or equal to, 3, and also that x cannot be smaller than, or equal to, -3. This condition simplifies to "x is between -3 and 3", which may be written as $-3 < x < 3$. So the correct answer is (d).

Method 2: Either $x \geq 0$ or $x \leq 0$. In the first case $|x| = x$, so $|x| < 3$ means $x < 3$, and, since $x \geq 0$, this implies $-3 < x < 3$. In the second case $|x| = -x$, so $|x| < 3$ means $-x < 3$, which simplifies to $x > -3$. Since $x \leq 0$, this implies $-3 < x < 3$ in this case as well. Converseley, suppose $-3 < x < 3$. Again either $x \geq 0$ or $x \leq 0$. If $x \geq 0$ then $|x| = x$ and $x < 3$ so $|x| < 3$. To handle the second case first multiply $-3 < x < 3$ by -1 to get $3 > -x > -3$. Hence, if $x \leq 0$, then $|x| = -x$ and $-x < 3$ so $|x| < 3$. So, in either case, $|x| < 3$. This shows that "$|x| < 3$" is equivalent to "$-3 < x < 3$" so the correct answer is (d).

7. The domain of the function

$f(x) = \dfrac{\sqrt{x-5}}{x}$

is the set of x satisfying

(a) $x \geq 5$
(b) $x < 5$ *and* $x \neq 0$
(c) $x > 0$
(d) $x \neq 0$ *and* $x \neq 5$
(e) none of the above

Answer: Saying that a number x is in the domain of f just means that the formula for $f(x)$ makes sense for that value of x. There are two potential problems with the formula: you can't take the square root of a negative number and you can't divide by 0. To avoid the first problem we must have $x - 5 \geq 0$, and this simplifies to $x \geq 5$. This condition also implies that x is not 0, so there is no problem with dividing by x. So the correct answer is (a).

8. $\dfrac{\sin(3x)}{\sin(2x)} =$

(a) $\frac{3}{2}$
(b) $\frac{3\pi}{2}$
(c) $\sin(x)$
(d) $\sin\left(\frac{3}{2}\right)$
(e) none of the above

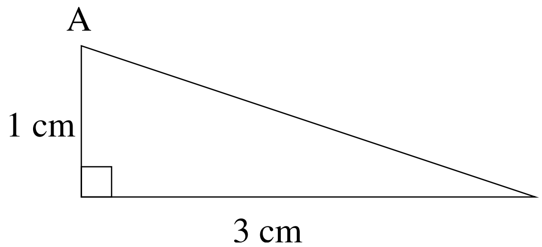
Answer: This problem is asking whether the given equation is an identity – and for this it must be true for all x for which the two sides of the equation make sense. If you go over all the trig identities that you know you will not find anything that is applicable. But this doesn't settle the issue, since there are many trig identities that you do not know.

In fact, the correct answer is (e): none of the above. The other possibilities can be ruled out by giving one or more examples to show that they do not work. The easiest example is $x = \pi/6$. Then $\sin(3x) = \sin(3 \cdot \pi/6) = \sin(\pi/2) = 1$ and $\sin(2x) = \sin(2 \cdot \pi/6) = \sin(\pi/3) = \sqrt{3}/2$, so

$$\frac{\sin(3x)}{\sin(2x)} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}.$$

This is not equal to the numbers in (a) or (b). Since $2/\sqrt{3}$ is greater than 1 it cannot be equal to the expressions in (c) or (d) since those expressions are values of the sine function, which can't be greater than 1.

9. For the right triangle below, find $\cos(A)$



(a) $\dfrac{1}{\sqrt{10}}$
(b) $\dfrac{3}{\sqrt{10}}$
(c) $\frac{1}{3}$
(d) 3
(e) none of the above

Answer: $\cos(A)$ is the ratio of the adjacent side to the hypoteneuse. By the Pythagorean Theorem the hypoteneuse is $\sqrt{1^2+3^2} = \sqrt{10}$, so

$$\cos(A) = \frac{1}{\sqrt{10}}$$

Hence the correct answer is (a).

10. Find the slope of the line $3x - 4y = 8$.
(a) $-\frac{8}{3}$
(b) $-\frac{3}{4}$
(c) $\frac{3}{4}$
(d) 3
(e) none of the above

Answer: Solve the equation of the line for y:

$$\begin{aligned} 3x - 4y &= 8 \\ -4y &= -3x + 8 \\ 4y &= 3x - 8 \\ y &= \frac{3}{4}x - 2 \end{aligned}$$

This is in the form $y = mx + b$ so the slope is $m = \dfrac{3}{4}$. Hence the correct answer is (c).