

The Operad of Locally Trivialized $G_{\mathbb{C}}$ -Bundles over Riemann Sphere and Generalized Affine Vertex Operator Algebras

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Abstract

For any connected and simply connected semi-simple Lie group G , let $G_{\mathbb{C}}$ be its complexification. With the sewing operations of principal $G_{\mathbb{C}}$ -bundles and the permutations of symmetric groups on the local trivializations, the moduli space $\mathcal{E}(G_{\mathbb{C}})$ of locally trivialized principal $G_{\mathbb{C}}$ -bundles over Riemann sphere forms an analytic partial operad.

First, we study the holomorphic operadic structure on the moduli spaces $\mathcal{E}(G_{\mathbb{C}})$ and their relations with the loop group $L_{an}G_{\mathbb{C}}$, the group $Diff^+(S^1)$ as well as the infinite dimensional Grassmannian. Secondly, we classify and construct explicitly all 1-dimensional modular functors $\tilde{\mathcal{E}}(G_{\mathbb{C}})$ over the moduli space $\mathcal{E}(G_{\mathbb{C}})$, including the determinant line bundle of $\tilde{\mathcal{E}}(G_{\mathbb{C}})$. Thirdly, we show that the category of the meromorphic algebras of 1-dimensional modular functor $\tilde{\mathcal{E}}(G_{\mathbb{C}})$ is isomorphic to the category of what we call the integrable generalized affine vertex operator algebras. This not only generalizes the remarkable theorem due to Frenkel and Huang about vertex operator algebras and the operad of Riemann sphere with punctures and local coordinates, but also gives some geometric interpretations for Frenkel and Zhu's algebraic constructions of affine vertex operator algebras.