SOCIETY OF ACTUARIES

EXAM MLC ACTUARIAL MODELS

EXAM MLC SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA examinations.

MLC-09-08 **PRINTED IN U.S.A.**

Question #1
Key: E

$${}_{2|}q_{\overline{30:34}} = {}_{2}p_{\overline{30:34}} - {}_{3}p_{\overline{30:34}}$$

$${}_{2}p_{30} = (0.9)(0.8) = 0.72$$

$${}_{2}p_{34} = (0.5)(0.4) = 0.20$$

$${}_{2}p_{30:34} = (0.72)(0.20) = 0.144$$

$${}_{2}p_{\overline{30:34}} = 0.72 + 0.20 - 0.144 = 0.776$$

$${}_{3}p_{30} = (0.72)(0.7) = 0.504$$

$${}_{3}p_{34} = (0.20)(0.3) = 0.06$$

$${}_{3}p_{30:34} = (0.504)(0.06) = 0.03024$$

$${}_{3}p_{\overline{30:34}} = 0.504 + 0.06 - 0.03024$$
$$= 0.53376$$

$${}_{2|}q_{\overline{30:34}} = 0.776 - 0.53376$$
$$= 0.24224$$

Alternatively,

$${}_{2|}q_{\overline{30:34}} = {}_{2|}q_{30} + {}_{2|}q_{34} - {}_{2|}q_{30:34}$$
$$= {}_{2}p_{30}q_{32} + {}_{2}p_{34}q_{36} - {}_{2}p_{30:34}(1 - p_{32:36})$$

= (0.9)(0.8)(0.3) + (0.5)(0.4)(0.7) – (0.9)(0.8)(0.5)(0.4) [1-(0.7)(0.3)]
= 0.216 + 0.140 – 0.144(0.79)
= 0.24224

Alternatively,

$${}_{2|}q_{\overline{30:34}} = {}_{3}q_{30} \times {}_{3}q_{34} - {}_{2}q_{30} \times {}_{2}q_{34}$$
$$= (1 - {}_{3}p_{30})(1 - {}_{3}p_{34}) - (1 - {}_{2}p_{30})(1 - {}_{2}p_{34})$$
$$= (1 - 0.504)(1 - 0.06) - (1 - 0.72)(1 - 0.20)$$
$$= 0.24224$$

(see first solution for ${}_{2}p_{30}$, ${}_{2}p_{34}$, ${}_{3}p_{30}$, ${}_{3}p_{34}$)

Question #2

Key: E

$$1000\bar{A}_x = 1000\left[\bar{A}^{1}_{x:\overline{10}|} + {}_{10|}\bar{A}_x\right]$$

$$= 1000\left[\int_0^{10} e^{-0.04t}e^{-0.06t}(0.06)dt + e^{-0.4}e^{-0.6}\int_0^{\infty} e^{-0.05t}e^{-0.07t}(0.07)dt\right]$$

$$= 1000\left[0.06\int_0^{10} e^{-0.1t}dt + e^{-1}(0.07)\int_0^{\infty} e^{-0.12t}dt\right]$$

$$= 1000\left[0.06\left[\frac{-e^{-0.10t}}{0.10}\right]_0^{10} + e^{-1}(0.07)\left[\frac{-e^{-0.12t}}{0.12}\right]_0^{\infty}\right]$$

$$= 1000\left[\frac{0.06}{0.10}\left[1-e^{-1}\right] + \frac{0.07}{0.12}e^{-1}\left[1-e^{-1.2}\right]\right]$$

$$= 1000(0.37927 + 0.21460) = 593.87$$

Because this is a timed exam, many candidates will know common results for constant force
and constant interest without integration.

For example $\bar{A}^{1}_{x:\overline{10}|} = \dfrac{\mu}{\mu+\delta}\left(1 - {}_{10}E_x\right)$

$${}_{10}E_x = e^{-10(\mu+\delta)}$$

$$\bar{A}_x = \frac{\mu}{\mu+\delta}$$

With those relationships, the solution becomes

$$1000\bar{A}_x = 1000\left[\bar{A}^{1}_{x:\overline{10}|} + {}_{10}E_x\ A_{x+10}\right]$$

$$= 1000\left[\left(\frac{0.06}{0.06+0.04}\right)\left(1-e^{-(0.06+0.04)10}\right) + e^{-(0.06+0.04)10}\left(\frac{0.07}{0.07+0.05}\right)\right]$$

$$= 1000\left[(0.60)\left(1-e^{-1}\right) + 0.5833e^{-1}\right]$$

$$= 593.86$$

Question #3
Key: D

$$E[Z] = \int_0^{\infty} b_t v^t\ {}_tp_x\ \mu(x+t)\ dt = \int_0^{\infty} e^{0.06\,t}e^{-0.08\,t}e^{-0.05\,t}\frac{1}{20}dt$$

$$= \frac{1}{20}\left(\frac{100}{7}\right)\left[-e^{-0.07t}\right]_0^{\infty} = \frac{5}{7}$$

$$E\left[Z^2\right]=\int_0^\infty \left(b_t v^t\right)^2 {}_t p_x \mu\left(x+t\right)dt=\int_0^\infty e^{0.12t}e^{-0.16t}e^{-0.05t}\,\frac{1}{20}dt=\frac{1}{20}\int_0^\infty e^{-0.09t}dt$$

$$=\frac{1}{20}\left(\frac{100}{9}\right)\left[e^{-0.09t}\right]_0^\infty=\frac{5}{9}$$

$$Var\left[Z\right]=\frac{5}{9}-\left(\frac{5}{7}\right)^2=0.04535$$

Question #4
Key: C

Let *ns* = nonsmoker and *s* = smoker

k =	$q_{x+k}^{(ns)}$	$p_{x+k}^{(ns)}$	$q_{x+k}^{(s)}$	$p_{x+k}^{(s)}$
0	.05	0.95	0.10	0.90
1	.10	0.90	0.20	0.80
2	.15	0.85	0.30	0.70

$$A_{x:\overline{2}|}^{1\,(ns)}= \quad v \quad q_x^{(ns)} + \quad v^2 \quad p_x^{(ns)} \quad q_{x+1}^{(ns)}$$

$$\frac{1}{1.02}\left(0.05\right) \quad \frac{1}{1.02^2} \quad 0.95\times 0.10=0.1403$$

$$A_{x:\overline{2}|}^{1\,(s)} \quad v \quad q_x^{(s)} \quad + \quad v^2 \quad p_x^{(s)} \quad q_{x+1}^{(s)}$$

$$\frac{1}{1.02} \quad \left(0.10\right)+\frac{1}{\left(1.02\right)^2} \quad 0.90\times 0.20=0.2710$$

$A_{x:\overline{2}|}^{1}=$ weighted average = (0.75)(0.1403) + (0.25)(0.2710)
= 0.1730

Question #5
Key: B

$$\mu_x^{(\tau)}=\mu_x^{(1)}+\mu_x^{(2)}+\mu_x^{(3)}=0.0001045$$

$${}_t p_x^{(\tau)}=e^{-0.0001045t}$$

$$\text{APV Benefits} = \int_0^\infty e^{-\delta t} 1{,}000{,}000 \, {}_t p_x^{(\tau)} \mu_x^{(1)} dt$$

$$+\int_0^\infty e^{-\delta t} 500{,}000 \, {}_t p_x^{(\tau)} \mu_x^{(2)} dt$$

$$+\int_0^\infty e^{-\delta \tau} 200{,}000 \, {}_t p_x^{(\tau)} \mu_x^{(3)} dt$$

$$= \frac{1{,}000{,}000}{2{,}000{,}000} \int_0^\infty e^{-0.0601045t} dt + \frac{500{,}000}{250{,}000} \int_0^\infty e^{-0.0601045t} dt + \frac{250{,}000}{10{,}000} \int_0^\infty e^{-0.0601045t} dt$$

$$= 27.5(16.6377) = 457.54$$

Question #6

Key: B

$$APV \text{ Benefits} = 1000 A^{1}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$

$$APV \text{ Premiums} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$

$$\text{Benefit premiums } \Rightarrow \text{Equivalence principle } \Rightarrow$$

$$1000 A^{1}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$

$$\pi = 1000 A^{1}_{40:\overline{20}|} / \ddot{a}_{40:\overline{20}|}$$

$$= \frac{161.32 - (0.27414)(369.13)}{14.8166 - (0.27414)(11.1454)}$$

$$= 5.11$$

While this solution above recognized that $\pi = 1000 P^{1}_{40:\overline{20}|}$ and was structured to take advantage of that, it wasn't necessary, nor would it save much time. Instead, you could do:

$$APV \text{ Benefits } = 1000 A_{40} = 161.32$$

$$APV \text{ Premiums } = \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} \sum_{k=0}^{\infty} {}_k E_{60} 1000 v q_{60+k}$$

$$= \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} 1000 A_{60}$$

$$= \pi\left[14.8166 - (0.27414)(11.1454)\right] + (0.27414)(369.13)$$

$$= 11.7612\pi + 101.19$$

$$11.7612\pi + 101.19 = 161.32$$

$$\pi = \frac{161.32 - 101.19}{11.7612} = 5.11$$

Question #7

Key: C

$$A_{70} = \frac{\delta}{i}\bar{A}_{70} = \frac{\ln(1.06)}{0.06}(0.53) = 0.5147$$

$$\ddot{a}_{70} = \frac{1-A_{70}}{d} = \frac{1-0.5147}{0.06/1.06} = 8.5736$$

$$\ddot{a}_{69} = 1 + vp_{69}\ddot{a}_{70} = 1 + \left(\frac{0.97}{1.06}\right)(8.5736) = 8.8457$$

$$\ddot{a}_{69}^{(2)} = \alpha(2)\ddot{a}_{69} - \beta(2) = (1.00021)(8.8457) - 0.25739$$
$$= 8.5902$$

Note that the approximation $\ddot{a}_x^{(m)} \cong \ddot{a}_x - \frac{(m-1)}{2m}$ works well (is closest to the exact answer, only off by less than 0.01). Since *m* = 2, this estimate becomes $8.8457 - \frac{1}{4} = 8.5957$

Question #8
Key: C

The following steps would do <u>in this multiple-choice context</u>:

1. From the answer choices, this is a recursion for an insurance or pure endowment.
2. Only C and E would satisfy $u(70) = 1.0$.
3. It is <u>not</u> E. The recursion for a pure endowment is simpler: $u(k) = \frac{1+i}{p_{k-1}}u(k-1)$
4. Thus, it must be C.

More rigorously, transform the recursion to its backward equivalent,
$u(k-1)$ in terms of $u(k)$:

$$u(k) = -\left(\frac{q_{k-1}}{p_{k-1}}\right) + \left(\frac{1+i}{p_{k-1}}\right)u(k-1)$$
$$p_{k-1}u(k) = -q_{k-1} + (1+i)\,u(k-1)$$
$$u(k-1) = vq_{k-1} + vp_{k-1}u(k)$$

This is the form of (a), (b) and (c) on page 119 of Bowers with $x = k-1$. Thus, the recursion could be:

$$A_x = vq_x + vp_x A_{x+1}$$

or $$A^{1}_{x:\overline{y-x}|} = vq_x + vp_x \; A^{\;1}_{x+1:\overline{y-x-1}|}$$

or $$A_{x:\overline{y-x}|} = vq_x + vp_x \; A_{x+1:\overline{y-x-1}|}$$

Condition (iii) forces it to be answer choice C

$u(k-1) = A_x$ fails at $x = 69$ since it is not true that

$$A_{69} = vq_{69} + (vp_{69})(1)$$

$u(k-1) = A^{1}_{x:\overline{y-x}|}$ fails at $x = 69$ since it is not true that

$$A^{1}_{69:\overline{1}|} = vq_{69} + (vp_{69})(1)$$

$u(k-1) = A_{x:\overline{y-x}|}$ is OK at $x = 69$ since

$$A_{69:\overline{1}|} = vq_{69} + (vp_{69})(1)$$

Note: While writing recursion in backward form gave us something exactly like page 119 of Bowers, in its original forward form it is comparable to problem 8.7 on page 251. Reasoning from that formula, with $\pi_h = 0$ and $b_{h+1} = 1$, should also lead to the correct answer.

Question #9
Key: A

You arrive first if <u>both</u> (A) the first train to arrive is a local <u>and</u> (B) no express arrives in the 12 minutes after the local arrives.

$P(A)=0.75$

Expresses arrive at Poisson rate of $(0.25)(20)=5$ per hour, hence 1 per 12 minutes.

$$f(0)=\frac{e^{-1}1^0}{0!}=0.368$$

A and B are independent, so

$P(A \text{ and } B)=(0.75)(0.368)=0.276$

Question #10

Key: E

d = 0.05 → *v* = 0.095

At issue

$$A_{40}=\sum_{k=0}^{49} v^{k+1}\,{}_{k|}q_{40}=0.02\left(v^1+...+v^{50}\right)=0.02v\left(1-v^{50}\right)/d=0.35076$$

$$\text{and } \ddot{a}_{40}=\left(1-A_{40}\right)/d=\left(1-0.35076\right)/0.05=12.9848$$

$$\text{so } P_{40}=\frac{1000A_{40}}{\ddot{a}_{40}}=\frac{350.76}{12.9848}=27.013$$

$$E\left({}_{10}L \,\middle|\, K(40)\geq 10\right)=1000A_{50}^{\text{Revised}}-P_{40}\ddot{a}_{50}^{\text{Revised}}=549.18-(27.013)(9.0164)=305.62$$

where

$$A_{50}^{\text{Revised}}=\sum_{k=0}^{24} v^{k+1}\,{}_{k|}q_{50}^{\text{Revised}}=0.04\left(v^1+...+v^{25}\right)=0.04v\left(1-v^{25}\right)/d=0.54918$$

$$\text{and } \ddot{a}_{50}^{\text{Revised}}=\left(1-A_{50}^{\text{Revised}}\right)/d=\left(1-0.54918\right)/0.05=9.0164$$

Question #11
Key: E

Let NS denote non-smokers and S denote smokers.

The shortest solution is based on the conditional variance formula

$$\operatorname{Var}(X) = E\left(\operatorname{Var}(X|Y)\right) + \operatorname{Var}\left(E(X|Y)\right)$$

Let Y = 1 if smoker; Y = 0 if non-smoker

$$E\left(\bar{a}_{\overline{T|}} | Y = 1\right) = \bar{a}_x^S = \frac{1 - \bar{A}_x^S}{\delta}$$

$$= \frac{1 - 0.444}{0.1} = 5.56$$

Similarly $E\left(\bar{a}_{\overline{T|}} | Y = 0\right) = \dfrac{1 - 0.286}{0.1} = 7.14$

$$E\left(E\left(\bar{a}_{\overline{T|}} | Y\right)\right) = E\left(E\left(\bar{a}_{\overline{T|}} | 0\right)\right) \times \text{Prob}(\text{Y=0}) + E\left(E\left(\bar{a}_{\overline{T|}} | 1\right)\right) \times \text{Prob}(\text{Y=1})$$

$$= (7.14)(0.70) + (5.56)(0.30)$$

$$= 6.67$$

$$E\left[\left(E\left(\bar{a}_{\overline{T|}} | Y\right)\right)^2\right] = \left(7.14^2\right)(0.70) + \left(5.56^2\right)(0.30)$$

$$= 44.96$$

$$\operatorname{Var}\left(E\left(\bar{a}_{\overline{T|}} | Y\right)\right) = 44.96 - 6.67^2 = 0.47$$

$$E\left(\operatorname{Var}\left(\bar{a}_{\overline{T|}} | Y\right)\right) = (8.503)(0.70) + (8.818)(0.30)$$

$$= 8.60$$

$$\operatorname{Var}\left(\bar{a}_{\overline{T|}}\right) = 8.60 + 0.47 = 9.07$$

Alternatively, here is a solution based on
$\operatorname{Var}(Y) = E\left(Y^2\right) - \left[E(Y)\right]^2$, a formula for the variance of any random variable. This can be
transformed into $E\left(Y^2\right) = \operatorname{Var}(Y) + \left[E(Y)\right]^2$ which we will use in its conditional form

$$E\left(\left(\bar{a}_{\overline{T|}}\right)^2 | \text{NS}\right) = \operatorname{Var}\left(\bar{a}_{\overline{T|}} | \text{NS}\right) + \left[E\left(\bar{a}_{\overline{T|}} | \text{NS}\right)\right]^2$$

$$\operatorname{Var}\left[\bar{a}_{\overline{T|}}\right] = E\left[\left(\bar{a}_{\overline{T|}}\right)^2\right] - \left(E\left[\bar{a}_{\overline{T|}}\right]\right)^2$$

$$E\left[\bar{a}_{\overline{T|}}\right] = E\left[\bar{a}_{\overline{T|}} | \text{S}\right] \times \text{Prob}[\text{S}] + E\left[\bar{a}_{\overline{T|}} | \text{NS}\right] \times \text{Prob}[\text{NS}]$$

$$= 0.30\bar{a}_x^{\text{S}} + 0.70\bar{a}_x^{\text{NS}}$$

$$= \frac{0.30\left(1-\bar{A}_x^{\text{S}}\right)}{0.1} + \frac{0.70\left(1-\bar{A}_x^{\text{NS}}\right)}{0.1}$$

$$= \frac{0.30(1-0.444)+0.70(1-0.286)}{0.1} = (0.30)(5.56)+(0.70)(7.14)$$

$$= 1.67 + 5.00 = 6.67$$

$$E\left[\left(\bar{a}_{\overline{T|}}\right)^2\right] = E\left[\bar{a}_{\overline{T|}}^{\,2} \middle| \text{S}\right]\times \text{Prob}[\text{S}] + E\left[\bar{a}_{\overline{T|}}^{\,2} \middle| \text{NS}\right]\times \text{Prob}[\text{NS}]$$

$$= 0.30\left(\text{Var}\left(\bar{a}_{\overline{T|}}\middle|\text{S}\right) + \left(E\left[\bar{a}_{\overline{T|}}\middle|\text{S}\right]\right)^2\right)$$

$$+0.70\left(\text{Var}\left(\bar{a}_{\overline{T|}}\middle|\text{NS}\right) + E\left(\bar{a}_{\overline{T|}}\middle|\text{NS}\right)^2\right)$$

$$= 0.30\left[8.818+(5.56)^2\right] + 0.70\left[8.503+(7.14)^2\right]$$

11.919 + 41.638 = 53.557

$$\text{Var}\left[\bar{a}_{\overline{T|}}\right] = 53.557-(6.67)^2 = 9.1$$

Alternatively, here is a solution based on $\bar{a}_{\overline{T|}} = \dfrac{1-v^T}{\delta}$

$$\text{Var}\left(\bar{a}_{\overline{T|}}\right) = \text{Var}\left(\frac{1}{\delta}-\frac{v^T}{\delta}\right)$$

$$= \text{Var}\left(\frac{-v^T}{\delta}\right) \text{ since } \text{Var}(X+\text{constant}) = \text{Var}(X)$$

$$= \frac{\text{Var}\left(v^T\right)}{\delta^2} \text{ since } \text{Var}(\text{constant}\times X) = \text{constant}^2\times\text{Var}(X)$$

$$= \frac{{}^2\bar{A}_x - \left(\bar{A}_x\right)^2}{\delta^2} \text{ which is Bowers formula 5.2.9}$$

This could be transformed into ${}^2A_x = \delta^2\,\text{Var}\left(\bar{a}_{\overline{T|}}\right) + \bar{A}_x^2$, which we will use to get ${}^2A_x^{\text{NS}}$ and ${}^2A_x^{\text{S}}$.

$$
\begin{aligned}
{}^2\bar{A}_x &= E\left[v^{2T}\right] \\
&= E\left[v^{2T} \mid \text{NS}\right] \times \text{Prob}(\text{NS}) + E\left[v^{2T} \mid \text{S}\right] \times \text{Prob}(\text{S}) \\
&= \left[\delta^2 \text{Var}\left(\bar{a}_{\overline{T|}} \mid \text{NS}\right) + \left(\bar{A}_x^{\text{NS}}\right)^2\right] \times \text{Prob}(\text{NS}) \\
&+ \left[\delta^2 \text{Var}\left(\bar{a}_{\overline{T|}} \mid \text{S}\right) + \left(\bar{A}_x^{\text{S}}\right)^2\right] \times \text{Prob}(\text{S}) \\
&= \left[(0.01)(8.503) + 0.286^2\right] \times 0.70 \\
&+ \left[(0.01)(8.818) + 0.444^2\right] \times 0.30 \\
&= (0.16683)(0.70) + (0.28532)(0.30) \\
&= 0.20238
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_x &= E\left[v^T\right] \\
&= E\left[v^T \mid \text{NS}\right] \times \text{Prob}(\text{NS}) + E\left[v^T \mid \text{S}\right] \times \text{Prob}(\text{S}) \\
&= (0.286)(0.70) + (0.444)(0.30) \\
&= 0.3334
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}\left(\bar{a}_{\overline{T|}}\right) &= \frac{{}^2\bar{A}_x - \left(\bar{A}_x\right)^2}{\delta^2} \\
&= \frac{0.20238 - 0.3334^2}{0.01} = 9.12
\end{aligned}
$$

Question #12
Key: A

To be a density function, the integral of *f* must be 1 (i.e., everyone dies eventually). The solution is written for the general case, with upper limit ∞. Given the distribution of $f_2(t)$, we could have used upper limit 100 here.

Preliminary calculations from the Illustrative Life Table:

$$
\frac{l_{50}}{l_0} = 0.8951
$$

$$
\frac{l_{40}}{l_0} = 0.9313
$$

$$1 = \int_0^\infty f_T(t)\,dt = \int_0^{50} k\, f_1(t)\,dt + \int_{50}^\infty 1.2\, f_2(t)dt$$

$$= k\int_0^{50} f_1(t)\,dt + 1.2\int_{50}^\infty f_2(t)dt$$

$$= k F_1(50) + 1.2\big(F_2(\infty) - F_2(50)\big)$$

$$= k\big(1 - {}_{50}p_0\big) + 1.2(1 - 0.5)$$

$$= k(1 - 0.8951) + 0.6$$

$$k = \frac{1 - 0.6}{1 - 0.8951} = 3.813$$

For $x \le 50, F_T(x) = \int_0^x 3.813 f_1(t)\,dt = 3.813 F_1(x)$

$$F_T(40) = 3.813\left(1 - \frac{l_{40}}{l_0}\right) = 3.813(1 - 0.9313) = 0.262$$

$$F_T(50) = 3.813\left(1 - \frac{l_{50}}{l_0}\right) = 3.813(1 - 0.8951) = 0.400$$

$${}_{10}p_{40} = \frac{1 - F_T(50)}{1 - F_T(40)} = \frac{1 - 0.400}{1 - 0.262} = 0.813$$

Question #13
Key: D

Let NS denote non-smokers, S denote smokers.

$$\text{Prob}(T < t) = \text{Prob}(T < t|\text{NS}) \times \text{Prob}(\text{NS}) + \text{Prob}(T < t|\text{S}) \times \text{Prob}(\text{S})$$

$$= \left(1 - e^{-0.1t}\right) \times 0.7 + \left(1 - e^{-0.2t}\right) \times 0.3$$

$$= 1 - 0.7e^{-0.1t} - 0.3e^{-0.2t}$$

$S(t) = 0.3e^{-0.2t} + 0.7e^{-0.1t}$

Want $\hat{t}$ such that $0.75 = 1 - S(\hat{t})$ or $0.25 = S(\hat{t})$

$0.25 = 0.3e^{-2\hat{t}} + 0.7e^{-0.1\hat{t}} = 0.3\left(e^{-0.1\hat{t}}\right)^2 + 0.7e^{-0.1\hat{t}}$

Substitute: let $x = e^{-0.1\hat{t}}$

$0.3x^2 + 0.7x - 0.25 = 0$

This is quadratic, so $x = \dfrac{-0.7 \pm \sqrt{0.49 + (0.3)(0.25)4}}{2(0.3)}$

$$x = 0.3147$$

$e^{-0.1\hat{t}} = 0.3147$ so $\hat{t} = 11.56$

Question #14
Key: A

$$\bar{P}\left(\bar{A}_x\right) = \mu = 0.03$$

$$^2\bar{A}_x = 0.20 = \frac{\mu}{2\delta + \mu} = \frac{0.03}{2\delta + 0.03}$$

$$\Rightarrow \delta = 0.06$$

$$\text{Var}\left({}_0L\right) = \frac{^2\bar{A}_x - \left(\bar{A}_x\right)^2}{\left(\delta\,\bar{a}\right)^2} = \frac{0.20 - \left(\frac{1}{3}\right)^2}{\left(\frac{0.06}{0.09}\right)^2} = 0.20$$

$$\text{where} \quad A = \frac{\mu}{\mu + \delta} = \frac{0.03}{0.09} = \frac{1}{3} \qquad \bar{a} = \frac{1}{\mu + \delta} = \frac{1}{0.09}$$

Question #15
Key: C

Let N = number of sales on that day
S = aggregate prospective loss at issue on those sales
K = curtate future lifetime

N~Poisson (0.2∗50) $\Rightarrow E[N] = \text{Var}[N] = 10$

${}_0L = 10{,}000v^{K+1} - 500\ddot{a}_{\overline{K+1|}}$ $\Rightarrow E\left[{}_0L\right] = 10{,}000A_{65} - 500\ddot{a}_{65}$

${}_0L = \left(10{,}000 + \dfrac{500}{d}\right)v^{K+1} - \dfrac{500}{d}$ $\Rightarrow Var\left[{}_0L\right] = \left(10{,}000 + \dfrac{500}{d}\right)^2 \left[{}^2A_{65} - \left(A_{65}\right)^2\right]$

$$S = {}_0L_1 + {}_0L_2 + \ldots + {}_0L_N$$

$$E[S] = E[N] \cdot E\left[{}_0L\right]$$

$$Var[S] = Var\left[{}_0L\right] \cdot E[N] + \left(E\left[{}_0L\right]\right)^2 \cdot \text{Var}[N]$$

$$\Pr(S<0)=\Pr\left(Z<\frac{0-E[S]}{\sqrt{Var[S]}}\right)$$

Substituting d = 0.06/(1+0.06), ${}^2A_{65}=0.23603,\ \ A_{65}=0.43980$ and $\ddot{a}_{65}=9.8969$ yields

$$E[{}_0L]=-550.45$$

$$\text{Var}[{}_0L]=15{,}112{,}000$$

$$E[S]=-5504.5$$

$$\text{Var}[S]=154{,}150{,}000$$

Std Dev (S) = 12,416

$$\Pr(S<0)=\Pr\left(\frac{\text{S}+5504.5}{12{,}416}<\frac{5504.5}{12{,}416}\right)$$

$$=\Pr(Z<0.443)$$

$$=0.67$$

With the answer choices, it was sufficient to recognize that:

$$0.6554=\Phi(0.4)<\Phi(0.443)<\Phi(0.5)=0.6915$$

By interpolation, $\Phi(0.443)\approx(0.43)\Phi(0.5)+(0.57)\Phi(0.4)$

$$=(0.43)(0.6915)+(0.57)(0.6554)$$

$$=0.6709$$

Question #16
Key: A

$$1000P_{40} = \frac{A_{40}}{\ddot{a}_{40}} = \frac{161.32}{14.8166} = 10.89$$

$$1000\,{}_{20}V_{40} = 1000\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}}\right) = 1000\left(1 - \frac{11.1454}{14.8166}\right) = 247.78$$

$${}_{21}V = \frac{\left({}_{20}V + 5000P_{40}\right)(1+i) - 5000q_{60}}{p_{60}}$$

$$= \frac{\left(247.78 + (5)(10.89)\right) \times 1.06 - 5000\left(0.01376\right)}{1 - 0.01376} = 255$$

[Note: For this insurance, ${}_{20}V = 1000\,{}_{20}V_{40}$ because retrospectively, this is identical to whole life]

Though it would have taken <u>much</u> longer, you can do this as a prospective reserve. The prospective solution is included for educational purposes, not to suggest it would be suitable under exam time constraints.

$1000P_{40} = 10.89$ as above

$$1000A_{40} + 4000\,{}_{20}E_{40}A^{1}_{60:\overline{5}|} = 1000P_{40} + 5000P_{40} \times {}_{20}E_{40}\,\ddot{a}_{60:\overline{5}|} + \pi\,{}_{20}E_{40} \times {}_{5}E_{60}\,\ddot{a}_{65}$$

where $A^{1}_{60:\overline{5}|} = A_{60} - {}_{5}E_{60}\,A_{65} = 0.06674$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40}\,\ddot{a}_{60} = 11.7612$$

$$\ddot{a}_{60:\overline{5}|} = \ddot{a}_{60} - {}_{5}E_{60}\,\ddot{a}_{65} = 4.3407$$

$$1000\left(0.16132\right) + \left(4000\right)\left(0.27414\right)\left(0.06674\right) =$$

$$= \left(10.89\right)\left(11.7612\right) + \left(5\right)\left(10.89\right)\left(0.27414\right)\left(4.3407\right) + \pi\left(0.27414\right)\left(0.68756\right)\left(9.8969\right)$$

$$\pi = \frac{161.32 + 73.18 - 128.08 - 64.79}{1.86544}$$

$$= 22.32$$

Having struggled to solve for π, you could calculate ${}_{20}V$ prospectively then (as above)
calculate ${}_{21}V$ recursively.

$${}_{20}V = 4000A^{1}_{60:\overline{5}|} + 1000A_{60} - 5000P_{40}\,\ddot{a}_{60:\overline{5}|} - \pi\,{}_{5}E_{60}\,\ddot{a}_{65}$$

$$= \left(4000\right)\left(0.06674\right) + 369.13 - \left(5000\right)\left(0.01089\right)\left(4.3407\right) - \left(22.32\right)\left(0.68756\right)\left(9.8969\right)$$

$$= 247.86 \text{ (minor rounding difference from } 1000\,{}_{20}V_{40}\text{)}$$

Or we can continue to ${}_{21}V$ prospectively

$${}_{21}V = 5000A^{1}_{61:\overline{4|}} + 1000\,{}_4E_{61}\,A_{65} - 5000P_{40}\,\ddot{a}_{61:\overline{4|}} - \pi\,{}_4E_{61}\,\ddot{a}_{65}$$

where ${}_4E_{61} = \dfrac{l_{65}}{l_{61}}v^4 = \left(\dfrac{7,533,964}{8,075,403}\right)(0.79209) = 0.73898$

$$A^{1}_{61:\overline{4|}} = A_{61} - {}_4E_{61}\,A_{65} = 0.38279 - 0.73898 \times 0.43980$$
$$= 0.05779$$
$$\ddot{a}_{61:\overline{4|}} = \ddot{a}_{61} - {}_4E_{61}\,\ddot{a}_{65} = 10.9041 - 0.73898 \times 9.8969$$
$$= 3.5905$$

$${}_{21}V = (5000)(0.05779) + (1000)(0.73898)(0.43980)$$
$$-(5)(10.89)(3.5905) - 22.32(0.73898)(9.8969)$$
$$= 255$$

Finally. A moral victory. Under exam conditions since prospective benefit reserves must equal retrospective benefit reserves, calculate whichever is simpler.

Question #17
Key: C

$$\mathrm{Var}(Z) = {}^2A_{41} - (A_{41})^2$$
$$A_{41} - A_{40} = 0.00822 = A_{41} - (vq_{40} + vp_{40}A_{41})$$
$$= A_{41} - (0.0028/1.05 + (0.9972/1.05)A_{41})$$
$$\Rightarrow A_{41} = 0.21650$$

$${}^2A_{41} - {}^2A_{40} = 0.00433 = {}^2A_{41} - \left(v^2q_{40} + v^2p_{40}\,{}^2A_{41}\right)$$
$$= {}^2A_{41} - (0.0028/1.05^2 + \left(0.9972/1.05^2\right)^2 A_{41})$$
$${}^2A_{41} = 0.07193$$

$$\mathrm{Var}(Z) = 0.07193 - 0.21650^2$$
$$= 0.02544$$

Question #18
Key: D

This solution looks imposing because there is no standard notation. Try to focus on the big picture ideas rather than starting with the details of the formulas.

Big picture ideas:

1. We can express the present values of the perpetuity recursively.
2. Because the interest rates follow a Markov process, the present value (at time t) of the future payments at time t depends only on the state you are in at time t, not how you got there.
3. Because the interest rates follow a Markov process, the present value of the future payments at times t_1 and t_2 are equal if you are in the same state at times t_1 and t_2.

Method 1: Attack without considering the special characteristics of this transition matrix.

Let $s_k =$ state you are in at time $k \left(\text{thus } s_k = 0, 1 \text{ or } 2\right)$

Let $Y_k =$ present value, at time k, of the future payments.
Y_k is a random variable because its value depends on the pattern of discount factors, which are random. The expected value of Y_k is not constant; it depends on what state we are in at time *k*.

Recursively we can write

$Y_k = v \times \left(1 + Y_{k+1}\right)$, where it would be better to have notation that indicates the *v*'s are not constant, but are realizations of a random variable, where the random variable itself has different distributions depending on what state we're in. However, that would make the notation so complex as to mask the simplicity of the relationship.

Every time we are in state 0 we have

$$E\left[Y_k \middle| s_k = 0\right] = 0.95 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 0\right]\right)$$

$$= 0.95 \times \left(1 + \left\{\left(E\left[Y_{k+1} \middle| s_{k+1} = 0\right]\right)\right) \times \text{Prob}\left(\text{s}_{\text{k+1}} = 0 \middle| s_k = 0\right]\right)$$

$$+ \left(E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right) \times \text{Prob}\left(s_{k+1} = 1 \middle| s_k = 0\right]\right)$$

$$= \left(E\left[Y_{k+1} \middle| s_{k+1} = 2\right]\right) \times \text{Prob}\left(s_{k+1} = 2 \middle| s_k = 0\right]\right)\right\}$$

$$= 0.95 \times \left(1 + E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right)$$

That last step follows because from the transition matrix if we are in state 0, we always move to state 1 one period later.

Similarly, every time we are in state 2 we have
$E\left[Y_k \middle| s_k = 2\right] = 0.93 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 2\right]\right)$
$\quad = 0.93 \times \left(1 + E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right)$
That last step follows because from the transition matrix if we are in state 2, we always move to state 1 one period later.

Finally, every time we are in state 1 we have
$E\left[Y_k \middle| s_k = 1\right] = 0.94 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 1\right]\right)$
$\quad = 0.94 \times \left(1 + \left\{E\left[Y_{k+1} \middle| s_{k+1} = 0\right] \times \Pr\left[s_{k+1} = 0 \middle| s_k = 1\right] + E\left[Y_{k+1} \middle| s_{k+1} = 2\right] \times \Pr\left[s_{k+1} = 2 \middle| s_k = 1\right]\right\}\right)$
$\quad = 0.94 \times \left(1 + \left\{E\left[Y_{k+1} \middle| s_{k+1} = 0\right] \times 0.9 + E\left[Y_{k+1} \middle| s_{k+1} = 2\right] \times 0.1\right\}\right)$. Those last two steps follow from the fact that from state 1 we always go to either state 0 (with probability 0.9) or state 2 (with probability 0.1).

Now let's write those last three paragraphs using this shorter notation: $x_n = E\left[Y_k \middle| s_k = n\right]$. We can do this because (big picture idea #3), the conditional expected value is only a function of the state we are in, not when we are in it or how we got there.

$x_0 = 0.95(1 + x_1)$
$x_1 = 0.94(1 + 0.9x_0 + 0.1x_2)$
$x_2 = 0.93(1 + x_1)$

That's three equations in three unknowns. Solve (by substituting the first and third into the second) to get $x_1 = 16.82$.

That's the answer to the question, the expected present value of the future payments given in state 1.

The solution above is almost exactly what we would have to do with <u>any</u> 3×3 transition matrix. As we worked through, we put only the non-zero entries into our formulas. But if for example the top row of the transition matrix had been $\begin{pmatrix}0.4 & 0.5 & 0.1\end{pmatrix}$, then the first of our three equations would have become $x_0 = 0.95(1 + 0.4x_0 + 0.5x_1 + 0.1x_2)$, similar in structure to our actual equation for x_1. We would still have ended up with three linear equations in three unknowns, just more tedious ones to solve.

Method 2: Recognize the patterns of changes for <u>this particular transition matrix</u>.

<u>This particular</u> transition matrix has a recurring pattern that leads to a much quicker solution. We are starting in state 1 and are guaranteed to be back in state 1 two steps later, with the same prospective value then as we have now.
Thus,

$$E[Y] = E[Y|\text{first move is to } 0] \times \Pr[\text{first move is to } 0] + E[Y|\text{first move is to } 2] \times \Pr[\text{first move is to } 2]$$

$$= 0.94 \times \left[\left(1 + 0.95 \times \left(1 + E[Y]\right)\right)\right] \times 0.9 + \left[0.94 \times \left[\left(1 + 0.93 \times \left(1 + E[Y]\right)\right) \times 0.1\right]\right.$$

(Note that the equation above is exactly what you get when you substitute x_0 and x_2 into the formula for x_1 in Method 1.)

$$= 1.6497 + 0.8037E[Y] + 0.1814 + 0.0874E[Y]$$

$$E[Y] = \frac{1.6497 + 0.1814}{(1 - 0.8037 - 0.0874)}$$

$$= 16.82$$

Question #19
Key: E

The number of problems solved in 10 minutes is Poisson with mean 2.
If she solves exactly one, there is 1/3 probability that it is #3.
If she solves exactly two, there is a 2/3 probability that she solved #3.
If she solves #3 or more, she got #3.

$f(0) = 0.1353$
$f(1) = 0.2707$
$f(2) = 0.2707$

$$P = \left(\frac{1}{3}\right)(0.2707) + \left(\frac{2}{3}\right)(0.2707) + (1 - 0.1353 - 0.2707 - 0.2707) = 0.594$$

Question #20
Key: D

$$\mu_x^{(\tau)} = \mu_x^{(1)}(t) + \mu_x^{(2)}(t)$$

$$= 0.2\mu_x^{(\tau)}(t) + \mu_x^{(2)}(t)$$

$$\Rightarrow \mu_x^{(2)}(t) = 0.8\mu_x^{(\tau)}(t)$$

$$q_x^{'(1)} = 1 - p_x^{'(1)} = 1 - e^{-\int_0^1 0.2k\, t^2\, dt} = 1 - e^{-0.2\frac{k}{3}} = 0.04$$

$$k/3 \Rightarrow \ln(1-0.04)/(-0.2) = 0.2041$$
$$k = 0.6123$$

$${}_2q_x^{(2)} = \int_0^2 {}_tp_x^{(\tau)}\mu_x^{(2)}\ dt = 0.8\int_0^2\ {}_tp_x^{(\tau)}\ \ \mu_x^{(\tau)}(t)\ dt$$

$$= 0.8\ {}_2q_x^{(\tau)} = 0.8\left(1 - {}_2p_x^{(\tau)}\right)$$

$${}_2p_x^{(\tau)} = e^{-\int_0^2 \mu_x(t)dt}$$

$$= e^{-\int_0^2 kt^2\, dt}$$

$$= e^{\frac{-8k}{3}}$$

$$= e^{\frac{-(8)(0.6123)}{3}}$$

$$= 0.19538$$

$${}_2q_x^{(2)} = 0.8(1 - 0.19538) = 0.644$$

Question #21
Key: A

k	$\min(k,3)$	$f(k)$	$f(k)\times(\min(k,3))$	$f(k)\times[\min(k,3)]^2$
0	0	0.1	0	0
1	1	(0.9)(0.2) = 0.18	0.18	0.18
2	2	(0.72)(0.3) = 0.216	0.432	0.864
3+	3	1-0.1-0.18-0.216 = 0.504	1.512	4.536
			2.124	5.580

$$E\left[\min(K,3)\right] = 2.124$$
$$E\left\{\left[\min\left(K,3\right)\right]^2\right\} = 5.580$$
$$Var\left[\min(K,3)\right] = 5.580 - 2.124^2 = 1.07$$

Note that $E\left[\min(K,3)\right]$ is the temporary curtate life expectancy, $e_{x:\overline{3|}}$ if the life is age *x*. Problem 3.17 in Bowers, pages 86 and 87, gives an alternative formula for the variance, basing the calculation on ${}_k p_x$ rather than ${}_{k|}q_x$.

Question #22
Key: B

$$s\left(60\right) = \frac{e^{-(0.1)(60)} + e^{-(0.08)(60)}}{2}$$
$$= 0.005354$$
$$s\left(61\right) = \frac{e^{-(0.1)(61)} + e^{-(0.08)(61)}}{2}$$
$$= 0.00492$$
$$q_{60} = 1 - \frac{0.00492}{0.005354} = 0.081$$

Question #23
Key: D

Let q_{64} for Michel equal the standard q_{64} plus *c*. We need to solve for *c*.
Recursion formula for a standard insurance:

$${}_{20}V_{45} = \left({}_{19}V_{45} + P_{45}\right)\left(1.03\right) - q_{64}\left(1 - {}_{20}V_{45}\right)$$

Recursion formula for Michel's insurance

$${}_{20}V_{45} = \left({}_{19}V_{45} + P_{45} + 0.01\right)\left(1.03\right) - \left(q_{64} + c\right)(1 - {}_{20}V_{45})$$

The values of ${}_{19}V_{45}$ and ${}_{20}V_{45}$ are the same in the two equations because we are told Michel's benefit reserves are the same as for a standard insurance.

Subtract the second equation from the first to get:

$$0 = -(1.03)(0.01) + c(1 - {}_{20}V_{45})$$

$$c = \frac{(1.03)(0.01)}{(1 - {}_{20}V_{45})}$$

$$= \frac{0.0103}{1 - 0.427}$$

$$= 0.018$$

Question #24
Key: B

K is the curtate future lifetime for one insured.
L is the loss random variable for one insurance.
L_{AGG} is the aggregate loss random variables for the individual insurances.
σ_{AGG} is the standard deviation of L_{AGG}.
M is the number of policies.

$$L = v^{K+1} - \pi \ddot{a}_{\overline{K+1}|} = \left(1 + \frac{\pi}{d}\right) v^{K+1} - \pi/d$$

$$E[L] = (A_x - \pi \ddot{a}_x) = A_x - \pi \frac{(1 - A_x)}{d}$$

$$= 0.24905 - 0.025\left(\frac{0.75095}{0.056604}\right) = -0.082618$$

$$\text{Var}[L] = \left(1 + \frac{\pi}{d}\right)^2 \left({}^2A_x - A_x^2\right) = \left(1 + \frac{0.025}{0.056604}\right)^2 \left(0.09476 - (0.24905)^2\right) = 0.068034$$

$$E[L_{AGG}] = M\,E[L] = -0.082618M$$

$$\text{Var}[L_{AGG}] = M\ \text{Var}[L] = M(0.068034) \Rightarrow \sigma_{AGG} = 0.260833\sqrt{M}$$

$$\Pr[L_{AGG} > 0] = \left[\frac{L_{AGG} - E[L_{AGG}]}{\sigma_{AGG}} > \frac{-E(L_{AGG})}{\sigma_{AGG}}\right]$$

$$\approx \Pr\left(N(0,1) > \frac{0.082618M}{\sqrt{M}(0.260833)}\right)$$

$$\Rightarrow 1.645 = \frac{0.082618\sqrt{M}}{0.260833}$$

$$\Rightarrow M = 26.97$$

$\Rightarrow$ minimum number needed = 27

Question #25
Key: D

Annuity benefit: $Z_1 = 12{,}000 \dfrac{1-v^{K+1}}{d}$ for $K = 0,1,2,...$

Death benefit: $Z_2 = Bv^{K+1}$ for $K = 0,1,2,...$

New benefit: $Z = Z_1 + Z_2 = 12{,}000 \dfrac{1-v^{K+1}}{d} + Bv^{K+1}$

$$= \frac{12{,}000}{d} + \left(B - \frac{12{,}000}{d} \right) v^{K+1}$$

$$\mathrm{Var}(Z) = \left(B - \frac{12{,}000}{d} \right)^2 \mathrm{Var}\left(v^{K+1} \right)$$

$$\mathrm{Var}(Z) = 0 \text{ if } B = \frac{12{,}000}{0.08} = 150{,}000 .$$

In the first formula for $\mathrm{Var}(Z)$, we used the formula, valid for any constants *a* and *b* and random variable *X*,

$$\mathrm{Var}(a + bX) = b^2 \mathrm{Var}(X)$$

Question #26
Key: A

$\mu_{xy}(t) = \mu_x(t) + \mu_y(t) = 0.08 + 0.04 = 0.12$

$\bar{A}_x = \mu_x(t) / \left(\mu_x(t) + \delta \right) = 0.5714$

$\bar{A}_y = \mu_y(t) / \left(\mu_y(t) + \delta \right) = 0.4$

$\bar{A}_{xy} = \mu_{xy}(t) / \left(\mu_{xy}(t) + \delta \right) = 0.6667$

$\bar{a}_{xy} = 1 / \left(\mu_{xy}(t) + \delta \right) = 5.556$

$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.5714 + 0.4 - 0.6667 = 0.3047$

Premium = 0.304762/5.556 = 0.0549

Question #27
Key: B

$$P_{40} = A_{40} / \ddot{a}_{40} = 0.16132 / 14.8166 = 0.0108878$$

$$P_{42} = A_{42} / \ddot{a}_{42} = 0.17636 / 14.5510 = 0.0121201$$

$$a_{45} = \ddot{a}_{45} - 1 = 13.1121$$

$$\begin{aligned} E\left[{}_3L \middle| K(42) \geq 3 \right] &= 1000A_{45} - 1000P_{40} - 1000P_{42}\, a_{45} \\ &= 201.20 - 10.89 - (12.12)(13.1121) \\ &= 31.39 \end{aligned}$$

Many similar formulas would work equally well. One possibility would be $1000\,{}_3V_{42} + (1000P_{42} - 1000P_{40})$, because prospectively after duration 3, this differs from the normal benefit reserve in that in the next year you collect $1000P_{40}$ instead of $1000P_{42}$.

Question #28
Key: E

$$E\left[\min(T,40)\right] = 40 - 0.005(40)^2 = 32$$

$$\begin{aligned} 32 &= \int_0^{40} t\, f(t)\,dt + \int_{40}^{w} 40 f(t)\,dt \\ &= \int_0^{w} t\, f(t)\,dt - \int_{40}^{w} t\, f(t)\,dt + 40(.6) \\ &= 86 - \int_{40}^{w} tf(t)\,dt \end{aligned}$$

$$\int_{40}^{w} tf(t)dt = 54$$

$$\mathring{e}_{40} = \frac{\int_{40}^{w}(t-40)\, f(t)\,dt}{s(40)} = \frac{54 - 40(.6)}{.6} = 50$$

Question #29
Key: B

$$d = 0.05 \Rightarrow v = 0.95$$

Step 1 Determine p_x from Kevin's work:

$$\begin{aligned} 608 + 350vp_x &= 1000vq_x + 1000v^2 p_x\left(p_{x+1} + q_{x+1}\right) \\ 608 + 350(0.95)\, p_x &= 1000(0.95)(1 - p_x) + 1000(0.9025)\, p_x(1) \\ 608 + 332.5\, p_x &= 950(1 - p_x) + 902.5\, p_x \\ p_x &= 342/380 = 0.9 \end{aligned}$$

Step 2 Calculate $1000P_{x:\overline{2}|}$, as Kira did:

$$608 + 350(0.95)(0.9) = 1000P_{x:\overline{2}|}\left[1 + (0.95)(0.9)\right]$$

$$1000P_{x:\overline{2}|} = \frac{[299.25 + 608]}{1.855} = 489.08$$

The first line of Kira's solution is that the actuarial present value of Kevin's benefit premiums is equal to the actuarial present value of Kira's, since each must equal the actuarial present value of benefits. The actuarial present value of benefits would also have been easy to calculate as

$$(1000)(0.95)(0.1) + (1000)\left(0.95^2\right)(0.9) = 907.25$$

Question #30
Key: E

Because no premiums are paid after year 10 for (x), ${}_{11}V_x = A_{x+11}$

Rearranging 8.3.10 from Bowers, we get ${}_{h+1}V = \dfrac{\left({}_hV + \pi_h\right)(1+i) - b_{h+1}q_{x+h}}{p_{x+h}}$

$$ {}_{10}V = \frac{(32,535 + 2,078)\times(1.05) - 100,000\times 0.011}{0.989} = 35,635.642 $$

$$ {}_{11}V = \frac{(35,635.642 + 0)\times(1.05) - 100,000\times 0.012}{0.988} = 36,657.31 = A_{x+11} $$

Question #31
Key: B

For De Moivre's law where $s(x) = \left(1 - \dfrac{x}{\omega}\right)$:

$$ \overset{\circ}{e}_x = \frac{\omega - x}{2} \text{ and } {}_tp_x = \left(1 - \frac{t}{\omega - x}\right) $$

$$ \overset{\circ}{e}_{45} = \frac{105-45}{2} = 30 $$

$$ \overset{\circ}{e}_{65} = \frac{105-65}{2} = 20 $$

$$ \overset{\circ}{e}_{45:65} = \int_0^{40} {}_tp_{45:65}\,dt = \int_0^{40} \frac{60-t}{60}\times\frac{40-t}{40}\,dt $$

$$ = \frac{1}{60\times 40}\left(60\times 40\times t - \frac{60+40}{2}t^2 + \frac{1}{3}t^3\right)\Bigg|_0^{40} $$

$$ = 15.56 $$

$$ \overset{\circ}{e}_{\overline{45:65}} = \overset{\circ}{e}_{45} + \overset{\circ}{e}_{65} - \overset{\circ}{e}_{45:65} $$

$$ = 30 + 20 - 15.56 = 34 $$

In the integral for $\overset{\circ}{e}_{45:65}$, the upper limit is 40 since 65 (and thus the joint status also) can survive a maximum of 40 years.

Question #32
Answer: E

$$\mu(4) = -s'(4)/s(4)$$

$$= \frac{-\left(-e^4/100\right)}{1-e^4/100}$$

$$= \frac{e^4/100}{1-e^4/100}$$

$$= \frac{e^4}{100-e^4}$$

$$= 1.202553$$

Question # 33
Answer: A

$$q_x^{(i)} = q_x^{(\tau)}\left[\frac{\ln\ p_x'^{(i)}}{\ln\ p_x^{(\tau)}}\right] = q_x^{(\tau)}\left[\frac{\ln\ e^{-\mu^{(i)}}}{\ln\ e^{-\mu^{(\tau)}}}\right]$$

$$= q_x^{(\tau)} \times \frac{\mu^{(i)}}{\mu^{(\tau)}}$$

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 1.5$$

$$q_x^{(\tau)} = 1 - e^{-\mu(\tau)} = 1 - e^{-1.5}$$

=0.7769

$$q_x^{(2)} = \frac{(0.7769)\mu^{(2)}}{\mu^{(\tau)}} = \frac{(0.5)(0.7769)}{1.5}$$

$$= 0.2590$$

Question # 34
Answer: D

$${}_{2|2}A_{[60]} = v^3 \quad \times \quad {}_2p_{[60]} \quad \times \quad q_{[60]+2} \quad +$$

↓	↓	↓
pay at end of year 3	live 2 years	then die in year 3

$$+v^4 \quad \times \quad {}_3p_{[60]} \quad \times \quad q_{60+3}$$

pay at end of year 4	live 3 years	then die in year 4

$$= \frac{1}{(1.03)^3}(1-0.09)(1-0.11)(0.13) + \frac{1}{(1.03)^4}(1-0.09)(1-0.11)(1-0.13)(0.15)$$

$$= 0.19$$

Question # 35
Answer: B

$$\bar{a}_x = \bar{a}_{x:\overline{5}|} + {}_5E_x\ \bar{a}_{x+5}$$

$$\bar{a}_{x:\overline{5}|} = \frac{1-e^{-0.07(5)}}{0.07} = 4.219 \text{ , where } 0.07 = \mu + \delta \text{ for } t < 5$$

$${}_5E_x = e^{-0.07(5)} = 0.705$$

$$\bar{a}_{x+5} = \frac{1}{0.08} = 12.5 \text{, where } 0.08 = \mu + \delta \text{ for } t \ge 5$$

$$\therefore \bar{a}_x = 4.219 + (0.705)(12.5) = 13.03$$

Question #36
Key: D

$$p_x^{(\tau)} = p'^{(1)}_x p'^{(2)}_x = 0.8(0.7) = 0.56$$

$$q_x^{(1)} = \left[\frac{\ln\left(p'^{(1)}_x\right)}{\ln\left(p_x^{(\tau)}\right)}\right] q_x^{(\tau)} \text{ since UDD in double decrement table}$$

$$= \left[\frac{\ln(0.8)}{\ln(0.56)}\right] 0.44$$

$$= 0.1693$$

$$_{0.3}q_{x+0.1}^{(1)} = \frac{0.3q_x^{(1)}}{1 - 0.1q_x^{(\tau)}} = 0.053$$

To elaborate on the last step:

$$_{0.3}q_{x+0.1}^{(1)} = \frac{\begin{pmatrix}\text{Number dying from cause} \\ \text{1 between } x+0.1 \text{ and } x+0.4\end{pmatrix}}{\text{Number alive at } x+0.1}$$

Since UDD in double decrement,

$$= \frac{l_x^{(\tau)}(0.3)q_x^{(1)}}{l_x^{(\tau)}\left(1 - 0.1q_x^{(\tau)}\right)}$$

Question #37
Key: E

$$\bar{P}\left(\bar{A}_x\right) = \frac{1}{\bar{a}_x} - \delta = \frac{1}{12} - 0.04 = 0.04333$$

$$
\begin{aligned}
{}_oL_e &= {}_oL + E \\
&= v^T - \bar{P}\left(\bar{A}_x\right)\bar{a}_{\overline{T|}} + c_o + (g-e)\bar{a}_{\overline{T|}} \\
&= v^T - \bar{P}\left(\bar{A}_x\right)\left(\frac{1-v^T}{\delta}\right) + c_o + (g-e)\left(\frac{1-v^T}{\delta}\right) \\
&= v^T\left(1 + \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} - \frac{(g-e)}{\delta}\right) - \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} + c_o + \frac{(g-e)}{\delta}
\end{aligned}
$$

$$Var\left({}_oL_e\right) = Var\left(v^T\right)\left(1 + \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} - \frac{(g-e)}{\delta}\right)^2$$

Above step is because for any random variable *X* and constants *a* and *b*, $Var(aX+b) = a^2 Var(X)$.

Apply that formula with $X = v^T$.

Plugging in,

$$
\begin{aligned}
Var\left({}_oL_e\right) &= (0.10)\left(1 + \frac{0.04333}{0.04} - \frac{(0.0030-0.0066)}{0.04}\right)^2 \\
&= (0.10)(2.17325)^2 \\
&= 0.472
\end{aligned}
$$

Question #38
Key: D

$$T = \begin{pmatrix} 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0 & 1 \end{pmatrix} \qquad T^2 = \begin{pmatrix} 0.52 & 0.13 & 0.35 \\ 0.39 & 0.39 & 0.22 \\ 0 & 0 & 1 \end{pmatrix}$$

Actuarial present value (A.P.V.) prem = 800(1 + (0.7 + 0.1) + (0.52 + 0.13)) = 1,960
A.P.V. claim = 500(1 + 0.7 + 0.52) + 3000(0 + 0.1 + 0.13) = 1800
Difference = 160

Question # 39
Answer: D

Per 10 minutes, find coins worth exactly 10 at Poisson rate $(0.5)(0.2)(10) = 1$

Per 10 minutes,

$$f(0) = 0.3679 \qquad F(0) = 0.3679$$
$$f(1) = 0.3679 \qquad F(1) = 0.7358$$
$$f(2) = 0.1839 \qquad F(2) = 0.9197$$
$$f(3) = 0.0613 \qquad F(3) = 0.9810$$

Let Period 1 = first 10 minutes; period 2 = next 10.

Method 1, succeed with 3 or more in period 1; or exactly 2, then one or more in period 2

$$P = (1 - F(2)) + f(2)(1 - F(0)) = (1 - 0.9197) + (0.1839)(1 - 0.3679)$$
$$= 0.1965$$

Method 2: fail in period 1 if < 2; $\Pr ob = F(1) = 0.7358$

fail in period 2 if exactly 2 in period 1, then 0; $\Pr ob = f(2) f(0)$

$$= (0.1839)(0.3679) = 0.0677$$

Succeed if fail neither period; $\Pr ob = 1 - 0.7358 - 0.0677$

$$= 0.1965$$

(Method 1 is attacking the problem as a stochastic process model; method 2 attacks it as a ruin model.)

Question # 40
Answer: D

Use Mod to designate values unique to this insured.

$$\ddot{a}_{60} = (1 - A_{60}) / d = (1 - 0.36933) / [(0.06) / (1.06)] = 11.1418$$

$$1000P_{60} = 1000A_{60} / \ddot{a}_{60} = 1000(0.36933 / 11.1418) = 33.15$$

$$A_{60}^{Mod} = v\left(q_{60}^{Mod} + p_{60}^{Mod} A_{61}\right) = \frac{1}{1.06}[0.1376 + (0.8624)(0.383)] = 0.44141$$

$$\ddot{a}^{Mod} = \left(1 - A_{60}^{Mod}\right) / d = (1 - 0.44141) / [0.06 / 1.06] = 9.8684$$

$$E\left[{}_0L^{Mod}\right] = 1000\left(A_{60}^{Mod} - P_{60}\ddot{a}_{60}^{Mod}\right)$$

$$= 1000[0.44141 - 0.03315(9.8684)]$$

$$= 114.27$$

Question # 41
Answer: D

The prospective reserve at age 60 per 1 of insurance is A_{60}, since there will be no future premiums. Equating that to the retrospective reserve per 1 of coverage, we have:

$$A_{60} = P_{40} \frac{\ddot{s}_{40:\overline{10}|}}{{}_{10}E_{50}} + P_{50}^{Mod}\ \ddot{s}_{50:\overline{10}|} - {}_{20}k_{40}$$

$$A_{60} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}\ {}_{10}E_{50}} + P_{50}^{Mod} \frac{\ddot{a}_{50:\overline{10}|}}{{}_{10}E_{50}} - \frac{A^{1}_{40:\overline{20}|}}{{}_{20}E_{40}}$$

$$0.36913 = \frac{0.16132}{14.8166} \times \frac{7.70}{(0.53667)(0.51081)} + P_{50}^{Mod} \frac{7.57}{0.51081} - \frac{0.06}{0.27414}$$

$$0.36913 = 0.30582 + 14.8196\ P_{50}^{Mod} - 0.21887$$

$$1000\ P_{50}^{Mod} = 19.04$$

Alternatively, you could equate the retrospective and prospective reserves at age 50. Your equation would be:

$$A_{50} - P_{50}^{Mod}\ \ddot{a}_{50:\overline{10}|} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}} - \frac{A^{1}_{40:\overline{10}|}}{{}_{10}E_{40}}$$

where $A^{1}_{40:\overline{10}|} = A_{40} - {}_{10}E_{40}\ A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$

$$= 0.02766$$

$$0.24905 - \left(P_{50}^{Mod}\right)(7.57) = \frac{0.16132}{14.8166} \times \frac{7.70}{0.53667} - \frac{0.02766}{0.53667}$$

$$1000 P_{50}^{Mod} = \frac{(1000)(0.14437)}{7.57} = 19.07$$

Alternatively, you could set the actuarial present value of benefits at age 40 to the actuarial present value of benefit premiums. The change at age 50 did not change the benefits, only the pattern of paying for them.

$$A_{40} = P_{40}\ \ddot{a}_{40:\overline{10}|} + P_{50}^{Mod}\ {}_{10}E_{40}\ \ddot{a}_{50:\overline{10}|}$$

$$0.16132 = \left(\frac{0.16132}{14.8166}\right)(7.70) + \left(P_{50}^{Mod}\right)(0.53667)(7.57)$$

$$1000 P_{50}^{Mod} = \frac{(1000)(0.07748)}{4.0626} = 19.07$$

Question # 42
Answer: A

$$d_x^{(2)} = q_x^{(2)} \times l_x^{(\tau)} = 400$$

$$d_x^{(1)} = 0.45(400) = 180$$

$$q_x'^{(2)} = \frac{d_x^{(2)}}{l_x^{(\tau)} - d_x^{(1)}} = \frac{400}{1000 - 180} = 0.488$$

$$p_x'^{(2)} = 1 - 0.488 = 0.512$$

Note: The UDD assumption was not critical except to have all deaths during the year so that 1000 - 180 lives are subject to decrement 2.

Question #43
Answer: D

Use "age" subscripts for years completed in program. E.g., p_0 applies to a person newly hired ("age" 0).

Let decrement 1 = fail, 2 = resign, 3 = other.

Then $q_0'^{(1)} = 1/4,\ q_1'^{(1)} = 1/5,\ q_2'^{(1)} = 1/3$

$q_0'^{(2)} = 1/5,\ q_1'^{(2)} = 1/3,\ q_2'^{(2)} = 1/8$

$q_0'^{(3)} = 1/10,\ q_1'^{(3)} = 1/9,\ q_2'^{(3)} = 1/4$

This gives $p_0^{(\tau)} = (1-1/4)(1-1/5)(1-1/10) = 0.54$

$p_1^{(\tau)} = (1-1/5)(1-1/3)(1-1/9) = 0.474$

$p_2^{(\tau)} = (1-1/3)(1-1/8)(1-1/4) = 0.438$

So $1_0^{(\tau)} = 200,\ 1_1^{(\tau)} = 200\,(0.54) = 108$, and $1_2^{(\tau)} = 108\,(0.474) = 51.2$

$q_2^{(1)} = \left[\log\, p_2'^{(1)} / \log\, p_2^{(\tau)}\right] q_2^{(\tau)}$

$q_2^{(1)} = \left[\log\left(\tfrac{2}{3}\right) / \log(0.438)\right]\left[1 - 0.438\right]$

$= (0.405/0.826)(0.562)$

$= 0.276$

$d_2^{(1)} = l_2^{(\tau)}\, q_2^{(1)}$

$= (51.2)(0.276) = 14$

Question #44
Answer: C

Let: N = number
X = profit
S = aggregate profit
subscripts G = "good", B = "bad", AB = "accepted bad"

$\lambda_G = \left(\tfrac{2}{3}\right)(60) = 40$

$\lambda_{AB} = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(60) = 10$ (If you have trouble accepting this, think instead of a heads-tails rule, that the application is accepted if the applicant's government-issued identification number, e.g. U.S. Social Security Number, is odd. It is not the same as saying he automatically alternates accepting and rejecting.)

$$Var(S_G) = E(N_G) \times Var(X_G) + Var(N_G) \times E(X_G)^2$$
$$= (40)(10{,}000) + (40)\left(300^2\right) = 4{,}000{,}000$$
$$Var(S_{AB}) = E(N_{AB}) \times Var(X_{AB}) + Var(N_{AB}) \times E(X_{AB})^2$$
$$= (10)(90{,}000) + (10)(-100)^2 = 1{,}000{,}000$$

S_G and S_{AB} are independent, so

$$Var(S) = Var(S_G) + Var(S_{AB}) = 4{,}000{,}000 + 1{,}000{,}000$$
$$= 5{,}000{,}000$$

If you don't treat it as three streams ("goods", "accepted bads", "rejected bads"), you can compute the mean and variance of the profit per "bad" received.

$\lambda_B = \left(\frac{1}{3}\right)(60) = 20$

If all "bads" were accepted, we would have $E\left(X^2{}_B\right) = Var(X_B) + E(X_B)^2$

$$= 90{,}000 + (-100)^2 = 100{,}000$$

Since the probability a "bad" will be accepted is only 50%,

$$E(X_B) = \text{Prob}(\text{accepted}) \times E\left(X_B \middle| \text{accepted}\right) + \text{Prob}(\text{not accepted}) \times E\left(X_B \middle| \text{not accepted}\right)$$
$$= (0.5)(-100) + (0.5)(0) = -50$$
$$E\left(X_B^2\right) = (0.5)(100{,}000) + (0.5)(0) = 50{,}000$$

Likewise,

Now $Var(S_B) = E(N_B) \times Var(X_B) + Var(N_B) \times E(X_B)^2$

$$= (20)(47{,}500) + (20)\left(50^2\right) = 1{,}000{,}000$$

S_G and S_B are independent, so

$$Var(S) = Var(S_G) + Var(S_B) = 4{,}000{,}000 + 1{,}000{,}000$$
$$= 5{,}000{,}000$$

Question #45
Key: E

For De Moivre's Law:

$$\overset{\circ}{e}_x = \frac{\omega - x}{2}$$

$$_{k|}q_x = \frac{1}{\omega - x}$$

$$A_x = \sum_{k=b}^{\omega-x-1} v^{k+1}\, _{k|}q_x = \frac{1}{\omega - x}\sum_{k=b}^{\omega-x-1} v^{k+1}$$

$$A_x = \frac{a_{\overline{\omega-x}|}}{\omega - x}$$

$$\ddot{a}_x = \frac{1 - A_x}{d}$$

$\overset{\circ}{e}_{50} = 25 \Rightarrow \omega = 100$ for typical annuitants

$\overset{\circ}{e}_y = 15 \Rightarrow y =$ Assumed age = 70

$$A_{70} = \frac{a_{\overline{30}|}}{30} = 0.45883$$
$$\ddot{a}_{70} = 9.5607$$
$$500000 = b\,\ddot{a}_{20} \Rightarrow b = 52,297$$

Question #46
Answer: B

$$\begin{aligned}{}_{10}E_{30:40} = {}_{10}p_{30}\ {}_{10}p_{40}\ v^{10} &= \left({}_{10}p_{30}v^{10}\right)\left({}_{10}p_{40}v^{10}\right)(1+i)^{10} \\ &= \left({}_{10}E_{30}\right)\left({}_{10}E_{40}\right)(1+i)^{10} \\ &= (0.54733)(0.53667)(1.79085) \\ &= 0.52604\end{aligned}$$

The above is only one of many possible ways to evaluate ${}_{10}p_{30}\ {}_{10}p_{40}\ v^{10}$, all of which should give 0.52604

$$\begin{aligned}a_{30:40:\overline{10}|} &= a_{30:40} - {}_{10}E_{30:40}\ a_{30+10:40+10} \\ &= \left(\ddot{a}_{30:40} - 1\right) - (0.52604)\left(\ddot{a}_{40:50} - 1\right) \\ &= (13.2068) - (0.52604)(11.4784) \\ &= 7.1687\end{aligned}$$

Question #47
Answer: A

Equivalence Principle, where π is annual benefit premium, gives

$$1000\left(A_{35} + (IA)_{35} \times \pi\right) = \ddot{a}_x \pi$$

$$\pi = \frac{1000\,A_{35}}{\left(\ddot{a}_{35} - (IA)_{35}\right)} = \frac{1000 \times 0.42898}{(11.99143 - 6.16761)}$$

$$= \frac{428.98}{5.82382}$$

$$= 73.66$$

We obtained $\ddot{a}_{35}$ from

$$\ddot{a}_{35} = \frac{1 - A_{35}}{d} = \frac{1 - 0.42898}{0.047619} = 11.99143$$

Question #48
Answer: C

Time until arrival = waiting time plus travel time.

Waiting time is exponentially distributed with mean $1/\lambda$. The time you may already have been waiting is irrelevant: exponential is memoryless.

You: E (wait) = $\frac{1}{20}$ hour = 3 minutes
E (travel) = $(0.25)(16)+(0.75)(28)=25$ minutes
E (total) = 28 minutes

Co-worker: E (wait) = $\frac{1}{5}$ hour = 12 minutes
E (travel) = 16 minutes
E (total) = 28 minutes

Question #49
Answer: C

$$\mu_{xy} = \mu_x + \mu_y = 0.14$$

$$\bar{A}_x = \bar{A}_y = \frac{\mu}{\mu+\delta} = \frac{0.07}{0.07+0.05} = 0.5833$$

$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy}+\delta} = \frac{0.14}{0.14+0.05} = \frac{0.14}{0.19} = 0.7368 \text{ and } \bar{a}_{xy} = \frac{1}{\mu_{xy}+\delta} = \frac{1}{0.14+0.05} = 5.2632$$

$$P = \frac{\bar{A}_{\overline{xy}}}{\bar{a}_{xy}} = \frac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_{xy}} = \frac{2(0.5833)-0.7368}{5.2632} = 0.0817$$

Question #50
Answer: E

$$\left({}_{20}V_{20} + P_{20}\right)(1+i) - q_{40}\left(1 - {}_{21}V_{20}\right) = {}_{21}V_{20}$$

$$(0.49 + 0.01)(1+i) - 0.022(1 - 0.545) = 0.545$$

$$(1+i) = \frac{(0.545)(1-0.022) + 0.022}{0.50}$$

$$= 1.11$$

$$\left({}_{21}V_{20} + P_{20}\right)(1+i) - q_{41}\left(1 - {}_{22}V_{20}\right) = {}_{22}V_{20}$$

$$(0.545 + .01)(1.11) - q_{41}(1 - 0.605) = 0.605$$

$$q_{41} = \frac{0.61605 - 0.605}{0.395}$$

$$= 0.028$$

Question #51
Answer: E

$$1000\ P_{60} = 1000\ A_{60} / \ddot{a}_{60}$$

$$= 1000\ v\left(q_{60} + p_{60} A_{61}\right) / \left(1 + p_{60}\ v\ \ddot{a}_{61}\right)$$

$$= 1000\left(q_{60} + p_{60}\ A_{61}\right) / \left(1.06 + p_{60}\ \ddot{a}_{61}\right)$$

$$= \left(15 + (0.985)(382.79)\right) / \left(1.06 + (0.985)(10.9041)\right) = 33.22$$

Question #52
Key: D

Since the rate of depletion is constant there are only 2 ways the reservoir can be empty sometime within the next 10 days.

Way #1:
There is no rainfall within the next 5 days

Way #2
There is one rainfall in the next 5 days
And it is a normal rainfall
And there are no further rainfalls for the next five days

Prob (Way #1) = Prob(0 in 5 days) = exp(-0.2*5) = 0.3679
Prob (Way #2) = Prob(1 in 5 days) × 0.8 × Prob(0 in 5 days)
= 5*0.2 exp(-0.2* 5)* 0.8 * exp(-0.2* 5)
= 1 exp(-1) * 0.8 * exp(-1) = 0.1083

Hence Prob empty at some time = 0.3679 + 0.1083 = 0.476

Question #53
Key: E

$$0.96 = e^{-(\mu_1+\lambda)}$$
$$\mu_1 + \lambda = -\ln(0.96) = 0.04082$$
$$\mu_1 = 0.04082 - \lambda = 0.04082 - 0.01 = 0.03082$$

Similarly

$$\mu_2 = -\ln(0.97) - \lambda = 0.03046 - 0.01 = 0.02046$$
$$\mu_{xy} = \mu_1 + \mu_2 + \lambda = 0.03082 + 0.02046 + 0.01 = 0.06128$$
$${}_5p_{xy} = e^{-(5)(0.06128)} = e^{-0.3064} = 0.736$$

Question #54
Answer: B

Transform these scenarios into a four-state Markov chain, where the final disposition of rates in any scenario is that they decrease, rather than if rates increase, as what is given.

State	from year t – 3 to year t – 2	from year t – 2 to year t – 1	Probability that year t will decrease from year t - 1
0	Decrease	Decrease	0.8
1	Increase	Decrease	0.6
2	Decrease	Increase	0.75
3	Increase	Increase	0.9

$$\text{Transition matrix is } \begin{bmatrix} 0.80 & 0.00 & 0.20 & 0.00 \\ 0.60 & 0.00 & 0.40 & 0.00 \\ 0.00 & 0.75 & 0.00 & 0.25 \\ 0.00 & 0.90 & 0.00 & 0.10 \end{bmatrix}$$

$$P_{00}^{2} + P_{01}^{2} = 0.8 * 0.8 + 0.2 * 0.75 = 0.79$$

For this problem, you don't need the full transition matrix. There are two cases to consider. Case 1: decrease in 2003, then decrease in 2004; Case 2: increase in 2003, then decrease in 2004.

For Case 1: decrease in 2003 (following 2 decreases) is 0.8; decrease in 2004 (following 2 decreases is 0.8. Prob(both) = 0.8×0.8 = 0.64
For Case 2: increase in 2003 (following 2 decreases) is 0.2; decrease in 2004 (following a decrease, then increase) is 0.75. Prob(both) = 0.2×0.75 = 0.15
Combined probability of Case 1 and Case 2 is 0.64 + 0.15 = 0.79

Question #55
Answer: B

$l_x = \omega - x = 105 - x$
$\Rightarrow {}_tP_{45} = l_{45+t} / l_{45} = 60 - t / 60$

Let K be the curtate future lifetime of (45). Then the sum of the payments is 0 if $K \le 19$ and is $K - 19$ if $K \ge 20$.

$${}_{20|}\ddot{a}_{45} = \sum_{K=20}^{60} 1 \times \left(\frac{60-K}{60}\right) \times 1$$

$$= \frac{(40+39+\ldots+1)}{60} = \frac{(40)(41)}{2(60)} = 13.6\overline{6}$$

Hence,

$$\text{Prob}\left(K - 19 > 13.6\overline{6}\right) = \text{Prob}\left(K > 32.6\overline{6}\right)$$

$$= \text{Prob}\left(K \ge 33\right) \text{ since } K \text{ is an integer}$$

$$= \text{Prob}\left(T \ge 33\right)$$

$$= {}_{33}p_{45} = \frac{l_{78}}{l_{45}} = \frac{27}{60}$$

$$= 0.450$$

Question #56
Answer: C

$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = 0.25 \rightarrow \mu = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = 0.4$$

$$\left(I\bar{A}\right)_x = \int_0^\infty {}_{s|}\bar{A}_x \, ds$$

$$\int_0^\infty {}_sE_x \, \bar{A}_x \, ds$$

$$= \int_0^\infty \left(e^{-0.1s}\right)(0.4)\, ds$$

$$= (0.4)\left(\frac{-e^{-0.1s}}{0.1}\right)\Bigg|_0^\infty = \frac{0.4}{0.1} = 4$$

Alternatively, using a more fundamental formula but requiring more difficult integration.

$$\left(I\bar{A}\right)_x = \int_0^\infty t \; {}_tp_x \, \mu_x(t) e^{-\delta t} dt$$

$$= \int_0^\infty t \, e^{-0.04t}(0.04)e^{-0.06t} dt$$

$$= 0.04\int_0^\infty t \, e^{-0.1t} dt$$

(integration by parts, not shown)

$$= 0.04\left(\frac{-t}{0.1} - \frac{1}{0.01}\right)e^{-0.1t}\Big|_0^\infty$$

$$= \frac{0.04}{0.01} = 4$$

Question #57
Answer: E

Subscripts A and B here just distinguish between the tools and do not represent ages.

We have to find $\overset{o}{e}_{\overline{AB}}$

$$\overset{o}{e}_A = \int_0^{10}\left(1-\frac{t}{10}\right)dt = t-\frac{t^2}{20}\bigg|_0^{10} = 10-5 = 5$$

$$\overset{o}{e}_B = \int_0^{7}\left(1-\frac{t}{7}\right)dt = t-\frac{t^2}{14}\bigg|_0^{7} = 49-\frac{49}{14} = 3.5$$

$$\overset{o}{e}_{AB} = \int_o^{7}\left(1-\frac{t}{7}\right)\left(1-\frac{t}{10}\right)dt = \int_0^{7}\left(1-\frac{t}{10}-\frac{t}{7}+\frac{t^2}{70}\right)dt$$

$$= t-\frac{t^2}{20}-\frac{t^2}{14}+\frac{t^3}{210}\bigg|_0^{7}$$

$$= 7-\frac{49}{20}-\frac{49}{14}+\frac{343}{210} = 2.683$$

$$\overset{o}{e}_{\overline{AB}} = \overset{o}{e}_A+\overset{o}{e}_B-\overset{o}{e}_{AB}$$

$$= 5+3.5-2.683 = 5.817$$

Question #58
Answer: A

$\mu_x^{(\tau)}(t) = 0.100 + 0.004 = 0.104$

${}_t p_x^{(\tau)} = e^{-0.104t}$

Actuarial present value (APV) = APV for cause 1 + APV for cause 2.

$$2000\int_0^5 e^{-0.04t} e^{-0.104t}(0.100)dt + 500{,}000\int_0^5 e^{-0.04t}\, e^{-0.104t}(0.400)dt$$

$$= \left(2000(0.10) + 500{,}000(0.004)\right)\int_0^5 e^{-0.144t}dt$$

$$= \frac{2200}{0.144}\left(1 - e^{-0.144(5)}\right) = 7841$$

Question #59
Answer: A

$R = 1 - p_x = q_x$

$S = 1 - p_x \times e^{(-k)}$ since $e^{-\int_0^1(\mu_x(t)+k)dt} = e^{-\int_0^1 \mu_x(t)dt - \int_0^1 k\, dt}$

$$= e^{-\int_0^1 \mu_x(t)dt} e^{-\int_0^1 k\, dt}$$

So $S = 0.75R \Rightarrow 1 - p_x \times e^{-k} = 0.75q_x$

$$e^{-k} = \frac{1 - 0.75q_x}{p_x}$$

$$e^k = \frac{p_x}{1 - 0.75q_x} = \frac{1 - q_x}{1 - 0.75q_x}$$

$$k = \ln\left[\frac{1 - q_x}{1 - 0.75q_x}\right]$$

Question #60
Key: C

$A_{60} = 0.36913 \qquad d = 0.05660$

${}^2A_{60} = 0.17741$

and $\sqrt{{}^2A_{60} - A_{60}^2} = 0.202862$

Expected Loss on one policy is $E\left[L(\pi)\right] = \left(100{,}000 + \frac{\pi}{d}\right)A_{60} - \frac{\pi}{d}$

Variance on one policy is $\text{Var}\left[L(\pi)\right] = \left(100{,}000 + \frac{\pi}{d}\right)^2 \left({}^2A_{60} - A_{60}^2\right)$

On the 10000 lives,

$E[S] = 10{,}000 E\left[L(\pi)\right]$ and $\text{Var}[S] = 10{,}000 \ \text{Var}\left[L(\pi)\right]$

The π is such that $0 - E[S]/\sqrt{\text{Var}[S]} = 2.326$ since $\Phi(2.326) = 0.99$

$$\frac{10{,}000\left(\frac{\pi}{d} - \left(100{,}000 + \frac{\pi}{d}\right)A_{60}\right)}{100\left(100{,}000 + \frac{\pi}{d}\right)\sqrt{{}^2A_{60} - A_{60}^2}} = 2.326$$

$$\frac{100\left(\frac{\pi}{d} - \left(100{,}000 + \frac{\pi}{d}\right)\right)(0.36913)}{\left(100{,}000 + \frac{\pi}{d}\right)(0.202862)} = 2.326$$

$$\frac{0.63087\frac{\pi}{d} - 36913}{100{,}000 + \frac{\pi}{d}} = 0.004719$$

$$0.63087\frac{\pi}{d} - 36913 = 471.9 = 0.004719\frac{\pi}{d}$$

$$\frac{\pi}{d} = \frac{36913 + 471.9}{0.63087 - 0.004719}$$

$$= 59706$$

$$\pi = 59706 \times d = 3379$$

Question #61
Key: C

$$ {}_1V = \left({}_0V + \pi\right)(1+i) - \left(1000 + {}_1V - {}_1V\right) \times q_{75} $$
$$ = 1.05\pi - 1000 q_{75} $$

Similarly,

$$ {}_2V = \left({}_1V + \pi\right) \times 1.05 - 1000 q_{76} $$
$$ {}_3V = \left({}_2V + \pi\right) \times 1.05 - 1000 q_{77} $$
$$ 1000 = {}_3V = \left(1.05^3 \pi + 1.05^2 \cdot \pi + 1.05\pi\right) - 1000 \times q_{75} \times 1.05^2 - 1000 \times 1.05 \times q_{76} - 1000 \times q_{77} \quad * $$
$$ \pi = \frac{1000 + 1000\left(1.05^2 q_{75} + 1.05 q_{76} + q_{77}\right)}{(1.05)^3 + (1.05)^2 + 1.05} $$
$$ = \frac{1000^x \left(1 + 1.05^2 \times 0.05169 + 1.05 \times 0.05647 + 0.06168\right)}{3.310125} $$
$$ = \frac{1000 \times 1.17796}{3.310125} = 355.87 $$

* This equation is algebraic manipulation of the three equations in three unknowns $\left({}_1V,\ {}_2V, \pi\right)$. One method – usually effective in problems where benefit = stated amount plus reserve, is to multiply the ${}_1V$ equation by 1.05^2, the ${}_2V$ equation by 1.05, and add those two to the ${}_3V$ equation: in the result, you can cancel out the ${}_1V$, and ${}_2V$ terms. Or you can substitute the ${}_1V$ equation into the ${}_2V$ equation, giving ${}_2V$ in terms of π, and then substitute that into the ${}_3V$ equation.

Question #62
Answer: D

$$ \bar{A}^{\,1}_{28:\overline{2}|} = \int_0^2 e^{-\delta t}\, {}^1\!/_{72}\, dt $$
$$ = \frac{1}{72\delta}\left(1 - e^{-2\delta}\right) = 0.02622 \text{ since } \delta = \ln(1.06) = 0.05827 $$
$$ \ddot{a}_{28:\overline{2}|} = 1 + v\left(\frac{71}{72}\right) = 1.9303 $$
$$ {}_3V = 500{,}000 \bar{A}^{\,1}_{28:\overline{2}|} - 6643\, \ddot{a}_{28:\overline{2}|} $$

= 287

Question #63
Answer: D

Let $\bar{A}_x$ and $\bar{a}_x$ be calculated with $\mu_x(t)$ and $\delta = 0.06$
Let $\bar{A}_x{}^*$ and $\bar{a}_x{}^*$ be the corresponding values with $\mu_x(t)$ increased by 0.03 and δ decreased by 0.03

$$\bar{a}_x = \frac{1-\bar{A}_x}{\delta} = \frac{0.4}{0.06} = 6.667$$

$$\bar{a}_x{}^* = \bar{a}_x$$

$$\left[\text{Proof: } \bar{a}_x{}^* = \int_0^\infty e^{-\int_0^t (\mu_x(s)+0.03)ds} e^{-0.03t} dt\right.$$

$$= \int_0^\infty e^{-\int_0^t \mu_x(s)ds} e^{-0.03t} e^{-0.03t} dt$$

$$= \int_0^\infty e^{-\int_0^t \mu_x(s)ds} e^{-0.06t} dt$$

$$\left. = \bar{a}_x \right]$$

$$\bar{A}_x{}^* = 1 - 0.03\bar{a}_x{}^* = 1 - 0.03\bar{a}_x$$

$$= 1 - (0.03)(6.667)$$

$$= 0.8$$

Question #64
Answer: A

	bulb ages				
Year	0	1	2	3	# replaced
0	10000	0	0	0	-
1	1000	9000	0	0	1000
2	100+2700	900	6300	0	2800
3	280+270+3150				3700

The diagonals represent bulbs that don't burn out.
E.g., of the initial 10,000, (10,000) (1-0.1) = 9000 reach year 1.
(9000) (1-0.3) = 6300 of those reach year 2.

Replacement bulbs are new, so they start at age 0.
At the end of year 1, that's (10,000) (0.1) = 1000
At the end of 2, it's (9000) (0.3) + (1000) (0.1) = 2700 + 100
At the end of 3, it's (2800) (0.1) + (900) (0.3) + (6300) (0.5) = 3700

Actuarial present value $= \frac{1000}{1.05} + \frac{2800}{1.05^2} + \frac{3700}{1.05^3}$

$= 6688$

Question #65
Key: E

$$\overset{\circ}{e}_{25:\overline{25}|} = \int_0^{15} {}_tp_{25}\,dt + {}_{15}p_{25}\int_0^{10} {}_tp_{40}\,dt$$

$$= \int_0^{15} e^{-.04t}\,dt + \left(e^{-\int_0^{15}.04\,ds}\right)\int_0^{10} e^{-.05t}\,dt$$

$$= \frac{1}{.04}\left(1-e^{-.60}\right) + e^{-.60}\left[\frac{1}{.05}\left(1-e^{-.50}\right)\right]$$

$$= 11.2797 + 4.3187$$

$$= 15.60$$

Question #66
Key: C

$${}_5p_{[60]+1} =$$

$$\left(1-q_{[60]+1}\right)\left(1-q_{[60]+2}\right)\left(1-q_{63}\right)\left(1-q_{64}\right)\left(1-q_{65}\right)$$

$$= (0.89)(0.87)(0.85)(0.84)(0.83)$$

$$= 0.4589$$

Question # 67
Key: E

$$12.50 = \bar{a}_x = \frac{1}{\mu + \delta} \Rightarrow \mu + \delta = 0.08 \Rightarrow \mu = \delta = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = 0.5$$

$$^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = \frac{1}{3}$$

$$\text{Var}\left(\bar{a}_{\overline{T|}}\right) = \frac{^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$$

$$= \frac{\frac{1}{3} - \frac{1}{4}}{0.0016} = 52.083$$

S.D. $= \sqrt{52.083} = 7.217$

Question # 68
Key: D

$$v = 0.90 \Rightarrow d = 0.10$$
$$A_x = 1 - d\ddot{a}_x = 1 - (0.10)(5) = 0.5$$

Benefit premium $\pi = \dfrac{5000A_x - 5000vq_x}{\ddot{a}_x}$

$$= \frac{(5000)(0.5) - 5000(0.90)(0.05)}{5} = 455$$

$$_{10}V_x = 1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}$$

$$0.2 = 1 - \frac{\ddot{a}_{x+10}}{5} \Rightarrow \ddot{a}_{x+10} = 4$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - (0.10)(4) = 0.6$$
$$_{10}V = 5000A_{x+10} - \pi\ddot{a}_{x+10} = (5000)(0.6) - (455)(4) = 1180$$

Question #69
Key: D

v is the lowest premium to ensure a zero % chance of loss in year 1 (The present value of the payment upon death is *v*, so you must collect at least *v* to avoid a loss should death occur).
Thus *v* = 0.95.

$$\mathrm{E}(\mathrm{Z}) = vq_x + v^2 p_x q_{x+1} = 0.95 \times 0.25 + (0.95)^2 \times 0.75 \times 0.2$$
$$= 0.3729$$
$$\mathrm{E}(\mathrm{Z}^2) = v^2 q_x + v^4 p_x q_{x+1} = (0.95)^2 \times 0.25 + (0.95)^4 \times 0.75 \times 0.2$$
$$= 0.3478$$

$$\mathrm{Var}(\mathrm{Z}) = \mathrm{E}(\mathrm{Z}^2) - (\mathrm{E}(\mathrm{Z}))^2 = 0.3478 - (0.3729)^2 = 0.21$$

Question #70
Key: D

Actuarial present value (APV) of future benefits =
$$= (0.005 \times 2000 + 0.04 \times 1000)/1.06 + (1 - 0.005 - 0.04)(0.008 \times 2000 + 0.06 \times 1000)/1.06^2$$
$$= 47.17 + 64.60$$
$$= 111.77$$

APV of future premiums $= \left[1 + (1 - 0.005 - 0.04)/1.06\right]50$
$$= (1.9009)(50)$$
$$= 95.05$$
$$E\left[{}_1L \middle| K(55) \geq 1\right] = 111.77 - 95.05 = 16.72$$

Question #71
Key: A

This is a nonhomogeneous Poisson process with intensity function

λ (t) = 3+3t, $0 \leq t \leq 2$, where *t* is time after noon

$$\text{Average } \lambda = \frac{\int_1^2 \lambda(t)\,dt}{1} = \int_1^2 (3+3t)\,dt$$

$$= \left[3t + \frac{3t^2}{2}\right]\Bigg|_1^2$$

$$= 7.5$$

$$f(2) = \frac{e^{-7.5}\ 7.5^2}{2!} = 0.0156$$

Question #72
Key: A

Let Z be the present value random variable for one life.
Let S be the present value random variable for the 100 lives.

$$E(Z) = 10\int_5^\infty e^{\delta t} e^{\mu t} \mu\, dt$$

$$= 10\frac{\mu}{\delta+\mu} e^{-(\delta+\mu)5}$$

$$= 2.426$$

$$E(Z^2) = 10^2\left(\frac{\mu}{2\delta+\mu}\right)e^{-(2\delta+\mu)5}$$

$$= 10^2\left(\frac{0.04}{0.16}\right)\left(e^{-0.8}\right) = 11.233$$

$$Var(Z) = E(Z^2) - \left(E(Z)\right)^2$$

$$= 11.233 - 2.426^2$$

$$= 5.348$$

$$\mathrm{E(S)} = 100\,\mathrm{E(Z)} = 242.6$$

$$\mathrm{Var(S)} = 100\ \mathrm{Var(Z)} = 534.8$$

$$\frac{F - 242.6}{\sqrt{534.8}} = 1.645 \rightarrow F = 281$$

Question #73
Key: D

Prob{only 1 survives} = 1-Prob{both survive}-Prob{neither survives}

$$= 1 - {}_3p_{50} \times {}_3p_{[50]} - \left(1 - {}_3p_{50}\right)\left(1 - {}_3p_{[50]}\right)$$

$$= 1 - \underbrace{(0.9713)(0.9698)(0.9682)}_{=0.912012}\underbrace{(0.9849)(0.9819)(0.9682)}_{0.936320} - (1 - 0.912012)(1 - 0.93632)$$

$$= 0.140461$$

Question # 74
Key: C

The tyrannosaur dies at the end of the first day if it eats no scientists that day. It dies at the end of the second day if it eats exactly one the first day and none the second day. If it does not die by the end of the second day, it will have at least 10,000 calories then, and will survive beyond 2.5.

Prob (dies) $= f(0) + f(1) f(0)$

$$= 0.368 + (0.368)(0.368)$$

$$= 0.503$$

since $f(0) = \frac{e^{-1}1^0}{0!} = 0.368$

$$f(1) = \frac{e^{-1}1^1}{1!} = 0.368$$

Question #75
Key: B

Let X = expected scientists eaten.

For each period, $E[X] = E[X|\text{dead}] \times \text{Prob}(\text{already dead}) + E[X|\text{alive}] \times \text{Prob}(\text{alive})$

$$= 0 \times \text{Prob}(\text{dead}) + E[X|\text{alive}] \times \text{Prob}(\text{alive})$$

Day 1, $\text{E}[X_1] = 1$

$$\text{Prob}(\text{dead at end of day 1}) = f(0) = \frac{e^{-1}0^1}{0!} = 0.368$$

Day 2, $\text{E}[X_2] = 0 \times 0.368 + 1 \times (1 - 0.368) = 0.632$

Prob (dead at end of day 2) = 0.503
[per problem 10]

Day 2.5, $\text{E}[X_{2.5}] = 0 \times 0.503 + 0.5 \times (1 - 0.503) = 0.249$

where $E\left[X_{2.5}|\text{alive}\right]=0.5$ since only $\frac{1}{2}$ day in period.

$$\mathrm{E}[X]=\mathrm{E}[X_1]+\mathrm{E}[X_2]+\mathrm{E}[X_{2.5}]=1+0.632+0.249=1.881$$
$$\mathrm{E}[10{,}000X]=18{,}810$$

Question # 76
Key: C

This solution applies the equivalence principle to each life. Applying the equivalence principle to the 100 life group just multiplies both sides of the first equation by 100, producing the same result for *P*.

$$APV(\text{Prems})=P=APV(\text{Benefits})=10q_{70}v+10\,p_{70}q_{71}v^2+Pp_{70}p_{71}v^2$$
$$P=\frac{(10)(0.03318)}{1.08}+\frac{(10)(1-0.03318)(0.03626)}{1.08^2}+\frac{P(1-0.03318)(1-0.03626)}{1.08^2}$$
$$=0.3072+0.3006+0.7988P$$
$$P=\frac{0.6078}{0.2012}=3.02$$

(*APV* above means Actuarial Present Value).

Question #77
Key: E

Level benefit premiums can be split into two pieces: one piece to provide term insurance
for *n* years; one to fund the reserve for those who survive.

Then,

$$P_x=P^1_{x:\overline{n|}}+P_{x:\overline{n|}}^{\ \ 1}\ {}_nV_x$$

And plug in to get

$$0.090=P^1_{x:\overline{n|}}+(0.00864)(0.563)$$
$$P^1_{x:\overline{n|}}=0.0851$$

Another approach is to think in terms of retrospective reserves. Here is one such solution:

$$\begin{aligned}
{}_nV_x &= \left(P_x - P^{1}_{x:\overline{n}|}\right)\ddot{s}_{x:\overline{n}|} \\
&= \left(P_x - P^{1}_{x:\overline{n}|}\right)\frac{\ddot{a}_{x:\overline{n}|}}{{}_nE_x} \\
&= \left(P_x - P^{1}_{x:\overline{n}|}\right)\frac{\ddot{a}_{x:\overline{n}|}}{P_{x:\overline{n}|}^{\ \ 1}\ddot{a}_{x:\overline{n}|}} \\
&= \frac{\left(P_x - P^{1}_{x:\overline{n}|}\right)}{\left(P_{x:\overline{n}|}^{\ \ 1}\right)}
\end{aligned}$$

$$0.563 = \left(0.090 - P^{1}_{x:\overline{n}|}\right) / 0.00864$$

$$\begin{aligned}
P^{1}_{x:\overline{n}|} &= 0.090 - (0.00864)(0.563) \\
&= 0.0851
\end{aligned}$$

Question #78
Key: A

$$\delta = \ln(1.05) = 0.04879$$

$$\begin{aligned}
\bar{A}_x &= \int_0^{\omega - x} {}_tp_x \mu_x(t) e^{-\delta t}\,dt \\
&= \int_0^{\omega - x} \frac{1}{\omega - x} e^{-\delta t}\,dt \ \text{ for DeMoivre} \\
&= \frac{1}{\omega - x}\bar{a}_{\overline{\omega - x}|}
\end{aligned}$$

From here, many formulas for ${}_{10}\bar{V}\left(\bar{A}_{40}\right)$ could be used. One approach is:

Since

$$\bar{A}_{50} = \frac{\bar{a}_{\overline{50|}}}{50} = \frac{18.71}{50} = 0.3742 \text{ so } \bar{a}_{50} = \left(\frac{1-\bar{A}_{50}}{\delta}\right) = 12.83$$

$$\bar{A}_{40} = \frac{\bar{a}_{\overline{60|}}}{60} = \frac{19.40}{60} = 0.3233 \text{ so } \bar{a}_{40} = \left(\frac{1-\bar{A}_{40}}{\delta}\right) = 13.87$$

$$\text{so } \bar{P}\left(\bar{A}_{40}\right) = \frac{0.3233}{13.87} = 0.02331$$

$$_{10}\bar{V}\left(\bar{A}_{40}\right) = \left[\bar{A}_{50} - \bar{P}\left(\bar{A}_{40}\right)\bar{a}_{50}\right] = \left[0.3742 - (0.02331)(12.83)\right] = 0.0751.$$

Question #79
Key: D

$$\bar{A}_x = E\left[v^{T(x)}\right] = E\left[v^{T(x)} \middle| NS\right] \times \text{Prob}(\text{NS}) + E\left[v^{T(x)} \middle| S\right] \times \text{Prob}(\text{S})$$

$$= \left(\frac{0.03}{0.03+0.08}\right) \times 0.70 + \left(\frac{0.6}{0.06+0.08}\right) \times 0.30$$

$$= 0.3195$$

Similarly, $^2\bar{A}_x = \left(\frac{0.03}{0.03+0.16}\right) \times 0.70 + \left(\frac{0.06}{0.06+0.16}\right) \times 0.30 = 0.1923.$

$$\text{Var}\left(\bar{a}_{\overline{T(x)|}}\right) = \frac{^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.1923 - 0.3195^2}{0.08^2} = 14.1.$$

Question #80
Key: B

$$_{2|}q_{\overline{80:84}} = {}_{2|}q_{80} + {}_{2|}q_{84} - {}_{2|}q_{80:84}$$

$$= 0.5 \times 0.4 \times (1-0.6) + 0.2 \times 0.15 \times (1-0.1)$$

= 0.10136

Using new p_{82} value of 0.3

$$0.5 \times 0.4 \times (1-0.3) + 0.2 \times 0.15 \times (1-0.1)$$

= 0.16118

Change = 0.16118 – 0.10136 = 0.06

Alternatively,

$_2p_{80} = 0.5 \times 0.4 = 0.20$

$_3p_{80} = {}_2p_{80} \times 0.6 = 0.12$

$_2p_{84} = 0.20 \times 0.15 = 0.03$

$_3p_{84} = {}_2p_{84} \times 0.10 = 0.003$

$_2p_{\overline{80:84}} = {}_2p_{80} + {}_2p_{84} - {}_2p_{80}\ {}_2p_{84}$ since independent

$= 0.20 + 0.03 - (0.20)(0.03) = 0.224$

$_3p_{\overline{80:84}} = {}_3p_{80} + {}_3p_{84} - {}_3p_{80}\ {}_3p_{84}$

$= 0.12 + 0.003 - (0.12)(0.003) = 0.12264$

$_{2|}q_{\overline{80:84}} = {}_2p_{\overline{80:84}} - {}_3p_{\overline{80:84}}$

$= 0.224 - 0.12264 = 0.10136$

Revised

$_3p_{80} = 0.20 \times 0.30 = 0.06$

$_3p_{\overline{80:84}} = 0.06 + 0.003 - (0.06)(0.003)$

$= 0.06282$

$_{2|}q_{\overline{80:84}} = 0.224 - 0.06282 = 0.16118$

change $= 0.16118 - 0.10136 = 0.06$

Question #81
Key: D

Poisson processes are separable. The aggregate claims process is therefore equivalent to two independent processes, one for Type I claims with expected frequency $\left(\frac{1}{3}\right)(3000) = 1000$ and
one for Type II claims.

Let $S_I =$ aggregate Type I claims.
$N_I =$ number of Type I claims.
$X_I =$ severity of a Type I claim (here = 10).

Since $X_I = 10$, a constant, $E(X_I) = 10$; $\text{Var}(X_I) = 0$.

$$\begin{aligned}\text{Var}(S_I) &= E(N_I)\ \text{Var}(X_I) + \text{Var}(N_I)\left[E(X_I)\right]^2 \\ &= (1000)(0) + (1000)(10)^2 \\ &= 100{,}000\end{aligned}$$

$$\begin{aligned}\text{Var}(S) &= \text{Var}(S_I) + \text{Var}(S_{II}) \text{ since independent} \\ 2{,}100{,}000 &= 100{,}000 + \text{Var}(S_{II}) \\ Var(S_{II}) &= 2{,}000{,}000\end{aligned}$$

Question #82
Key: A

$$\begin{aligned}{}_5p_{50}^{(\tau)} &= {}_5p_{50}'^{(1)}\ {}_5p_{50}'^{(2)} \\ &= \left(\frac{100-55}{100-50}\right)e^{-(0.05)(5)} \\ &= (0.9)(0.7788) = 0.7009\end{aligned}$$

Similarly

$$\begin{aligned}{}_{10}p_{50}^{(\tau)} &= \left(\frac{100-60}{100-50}\right)e^{-(0.05)(10)} \\ &= (0.8)(0.6065) = 0.4852\end{aligned}$$

$$\begin{aligned}{}_{5|5}q_{50}^{(\tau)} = {}_5p_{50}^{(\tau)} - {}_{10}p_{50}^{(\tau)} &= 0.7009 - 0.4852 \\ &= 0.2157\end{aligned}$$

Question #83
Key: C

Only decrement 1 operates before t = 0.7

${}_{0.7}q_{40}'^{(1)} = (0.7)\,q_{40}'^{(1)} = (0.7)(0.10) = 0.07$ since UDD

Probability of reaching t = 0.7 is 1-0.07 = 0.93

Decrement 2 operates only at t = 0.7, eliminating 0.125 of those who reached 0.7

$q_{40}^{(2)} = (0.93)(0.125) = 0.11625$

Question #84
Key: C

$$\pi\left(1+{}_2p_{80}v^2\right)=1000A_{80}+\frac{\pi v q_{80}}{2}+\frac{\pi v^3{}_2p_{80}q_{82}}{2}$$

$$\pi\left(1+\frac{0.83910}{1.06^2}\right)=665.75+\pi\left(\frac{0.08030}{2(1.06)}+\frac{0.83910\times 0.09561}{2(1.06)^3}\right)$$

$$\pi(1.74680)=665.75+\pi(0.07156)$$
$$\pi(1.67524)=665.75$$
$$\pi=397.41$$

Where ${}_2p_{80}=\dfrac{3,284,542}{3,914,365}=0.83910$

Or ${}_2p_{80}=(1-0.08030)(1-0.08764)=0.83910$

Question #85
Key: E

At issue, actuarial present value (APV) of benefits

$$=\int_0^\infty b_t v^t{}_tp_{65}\,\mu_{65}(t)dt$$
$$=\int_0^\infty 1000\left(e^{0.04t}\right)\left(e^{-0.04t}\right){}_tp_{65}\,\mu_{65}(t)dt$$
$$=1000\int_0^\infty {}_tp_{65}\;\mu_{65}(t)dt=1000\;{}_\infty q_{65}=1000$$

APV of premiums $=\pi\,\bar{a}_{65}=\pi\left(\dfrac{1}{0.04+0.02}\right)=16.667\pi$

Benefit premium $\pi=1000/16.667=60$

$${}_2\bar{V}=\int_0^\infty b_{2+u}\;v^u{}_up_{67}\;\mu_{65}(2+u)du-\pi\,\bar{a}_{67}$$
$$=\int_0^\infty 1000\,e^{0.04(2+u)}e^{-0.04u}\;{}_up_{67}\;\mu_{65}(2+u)du-(60)(16.667)$$
$$=1000e^{0.08}\int_0^\infty {}_up_{67}\mu_{65}(2+u)\,du-1000$$
$$=1083.29\;{}_\infty q_{67}-1000=1083.29-1000=83.29$$

Question #86
Key: B

(1) $a_{x:\overline{20}|} = \ddot{a}_{x:\overline{20}|} - 1 + {}_{20}E_x$

(2) $\ddot{a}_{x:\overline{20}|} = \dfrac{1 - A_{x:\overline{20}|}}{d}$

(3) $A_{x:\overline{20}|} = A^{1}_{x:\overline{20}|} + A_{x:\overset{\ \ 1}{\overline{20}|}}$

(4) $A_x = A^{1}_{x:\overline{20}|} + {}_{20}E_x\ A_{x+20}$

$$0.28 = A^{1}_{x:\overline{20}|} + (0.25)(0.40)$$
$$A^{1}_{x:\overline{20}|} = 0.18$$

Now plug into (3): $A_{x:\overline{20}|} = 0.18 + 0.25 = 0.43$

Now plug into (2): $\ddot{a}_{x:\overline{20}|} = \dfrac{1-0.43}{(0.05/1.05)} = 11.97$

Now plug into (1): $a_{x:\overline{20}|} = 11.97 - 1 + 0.25 = 11.22$

Question #87
Key: A

$$p_1 = \int_0^\infty p(1|\lambda) f(\lambda)\, d\lambda = \int_0^\infty \frac{e^{-\lambda}\lambda^1}{1!} \frac{(\lambda/2)e^{-(\lambda/2)}}{\lambda\Gamma(1)} d\lambda$$
$$= \frac{1}{2}\int_0^\infty \lambda e^{-\frac{3\lambda}{2}} d\lambda$$

[Integrate by parts; not shown]

$$= \frac{1}{2}\left(-\frac{2}{3}\lambda e^{-\frac{3}{2}\lambda} - \frac{4}{9}e^{-\frac{3}{2}\lambda}\right)\Bigg|_0^\infty$$
$$= \frac{2}{9} = 0.22$$

Question #88
Key: B

$$e_x = p_x + p_x e_{x+1} \Rightarrow p_x = \frac{e_x}{1+e_{x+1}} = \frac{8.83}{9.29} = 0.95048$$

$$\ddot{a}_x = 1 + vp_x + v^2\,{}_2p_x + ...$$

$$\ddot{a}_{x:\overline{2}|} = 1 + v + v^2\,{}_2p_x + ...$$

$$\ddot{a}_{x:\overline{2}|} - \ddot{a}_x = vq_x = 5.6459 - 5.60 = 0.0459$$

$$v(1-0.95048) = 0.0459$$

$$v = 0.9269$$

$$i = \frac{1}{v} - 1 = 0.0789$$

Question #89
Key: E

$$M = \text{Initial state matrix } = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}$$

$$T = \text{One year transition matrix } = \begin{bmatrix}0.20 & 0.80 & 0 & 0 \\ 0.50 & 0 & 0.50 & 0 \\ 0.75 & 0 & 0 & 0.25 \\ 1.00 & 0 & 0 & 0\end{bmatrix}$$

$$M \times T = \begin{bmatrix}0.20 & 0.80 & 0 & 0\end{bmatrix}$$

$$(M \times T) \times T = \begin{bmatrix}0.44 & 0.16 & 0.40 & 0\end{bmatrix}$$

$$((M \times T) \times T) \times T = \begin{bmatrix}0.468 & 0.352 & 0.08 & 0.10\end{bmatrix}$$

Probability of being in state *F* after three years = 0.468.

$$\text{Actuarial present value} = (0.468v^3)(500) = 171$$

Note:
Only the first entry of the last matrix need be calculated (verifying that the four sum to 1 is useful “quality control.”)

Question #90
Key: B

Let Y_i be the number of claims in the ith envelope.

Let $X(13)$ be the aggregate number of claims received in 13 weeks.

$$E[Y_i] = (1\times 0.2) + (2\times 0.25) + (3\times 0.4) + (4\times 0.15) = 2.5$$
$$E\left[Y_i^2\right] = (1\times 0.2) + (4\times 0.25) + (9\times 0.4) + (16\times 0.15) = 7.2$$
$$E[X(13)] = 50\times 13\times 2.5 = 1625$$
$$\text{Var}[X(13)] = 50\times 13\times 7.2 = 4680$$
$$\text{Prob}\{\text{X}(13)\le Z\} = 0.90 = \Phi(1.282)$$
$$\Rightarrow \text{Prob}\left\{\frac{X(13)-1625}{\sqrt{4680}}\le 1.282\right\}$$
$$X(13)\le 1712.7$$

Note: The formula for $\text{Var}[X(13)]$ took advantage of the frequency's being Poisson.
The more general formula for the variance of a compound distribution, $\text{Var}(S) = E(N)\ \text{Var}(X) + \text{Var}(N)E(X)^2$, would give the same result.

Question #91
Key: E

$$\mu^M(60) = \frac{1}{\omega-60} = \frac{1}{75-60} = \frac{1}{15}$$
$$\mu^F(60) = \frac{1}{\omega'-60} = \frac{1}{15}\times\frac{3}{5} = \frac{1}{25}\Rightarrow \omega' = 85$$
$${}_tp_{65}^M = 1-\frac{t}{10}$$
$${}_tp_{60}^F = 1-\frac{t}{25}$$

Let *x* denote the male and *y* denote the female.

$\overset{\circ}{e}_x = 5$ (mean for uniform distribution over $(0,10)$)

$\overset{\circ}{e}_y = 12.5$ (mean for uniform distribution over $(0,25)$)

$$\overset{\circ}{e}_{xy} = \int_0^{10}\left(1-\frac{t}{10}\right)\left(1-\frac{t}{25}\right)\cdot dt$$

$$= \int_0^{10}\left(1-\frac{7}{50}t+\frac{t^2}{250}\right)\cdot dt$$

$$= \left(t-\frac{7}{100}t^2+\frac{t^3}{750}\right)\Bigg|_0^{10} = 10-\frac{7}{100}\times 100+\frac{1000}{750}$$

$$= 10-7+\frac{4}{3} = \frac{13}{3}$$

$$\overset{\circ}{e}_{\overline{xy}} = \overset{\circ}{e}_x+\overset{\circ}{e}_y-\overset{\circ}{e}_{xy} = 5+\frac{25}{2}-\frac{13}{3} = \frac{30+75-26}{6} = 13.17$$

Question #92
Key: B

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = \frac{1}{3}$$

$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{1}{5}$$

$$\bar{P}\left(\bar{A}_x\right) = \mu = 0.04$$

$$\text{Var}(L) = \left(1+\frac{\bar{P}\left(\bar{A}_x\right)}{\delta}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$= \left(1+\frac{0.04}{0.08}\right)^2\left(\frac{1}{5}-\left(\frac{1}{3}\right)^2\right)$$

$$= \left(\frac{3}{2}\right)^2\left(\frac{4}{45}\right)$$

$$= \frac{1}{5}$$

Question #93
Key: A

Let π be the benefit premium
Let ${}_kV$ denote the benefit reserve a the end of year *k*.
For any $n, \left({}_nV+\pi\right)(1+i)=\left(q_{25+n}\times{}_{n+1}V+p_{25+n}\times{}_{n+1}V\right)$

$$={}_{n+1}V$$

Thus ${}_1V=\left({}_0V+\pi\right)(1+i)$

$${}_2V=\left({}_1V+\pi\right)(1+i)=\left(\pi(1+i)+\pi\right)(1+i)=\pi\ddot{s}_{\overline{2}|}$$
$${}_3V=\left({}_2V+\pi\right)(1+i)=\left(\pi\ddot{s}_{\overline{2}|}+\pi\right)(1+i)=\pi\ddot{s}_{\overline{3}|}$$

By induction (proof omitted)

$${}_nV=\pi\ddot{s}_{\overline{n}|}$$

For $n=35,\ {}_nV=\ddot{a}_{60}$ (actuarial present value of future benefits; there are no future premiums)

$$\ddot{a}_{60}=\pi\ \ddot{s}_{\overline{35}|}$$
$$\pi=\frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35}|}}$$

For $n=20,\quad {}_{20}V=\pi\ \ddot{s}_{\overline{20}|}$

$$=\left(\frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35}|}}\right)\ddot{s}_{\overline{20}|}$$

Alternatively, as above

$$\left({}_nV+\pi\right)(1+i)={}_{n+1}V$$

Write those equations, for $n=0$ to $n=34$

$$0:\left({}_0V+\pi\right)(1+i)={}_1V$$
$$1:\left({}_1V+\pi\right)(1+i)={}_2V$$
$$2:\left({}_2V+\pi\right)(1+i)={}_3V$$
$$\vdots$$
$$34:\left({}_{34}V+\pi\right)(1+i)={}_{35}V$$

Multiply equation *k* by $(1+i)^{34-k}$ and sum the results:

$$\left({}_0V+\pi\right)(1+i)^{35}+\left({}_1V+\pi\right)(1+i)^{34}+\left({}_2V+\pi\right)(1+i)^{33}+\cdots+\left({}_{34}V+\pi\right)(1+i)=$$
$${}_1V(1+i)^{34}+{}_2V(1+i)^{33}+{}_3V(1+i)^{32}+\cdots+{}_{34}V(1+i)+{}_{35}V$$

For $k=1,2,\cdots,34,$ the ${}_kV(1+i)^{35-k}$ terms in both sides cancel, leaving

$${}_0V(1+i)^{35}+\pi\left[(1+i)^{35}+(1+i)^{34}+\cdots+(1+i)\right]={}_{35}V$$

Since ${}_0V=0$

$$\pi\,\ddot{s}_{\overline{35}|}={}_{35}V$$

$$=\ddot{a}_{60}$$

(see above for remainder of solution)

Question #94
Key: B

$$\mu_{\overline{xy}}(t)=\frac{{}_tq_y\;{}_tp_x\mu(x+t)+{}_tq_x\;{}_tp_y\;\mu(y+t)}{{}_tq_x\times\;{}_tp_y+{}_tp_x\times{}_tq_y+{}_tp_x\times{}_tp_y}$$

For (*x*) = (*y*) = (50)

$$\mu_{\overline{50:50}}(10.5)=\frac{({}_{10.5}q_{50})({}_{10}p_{50})q_{60}\cdot 2}{({}_{10.5}q_{50})({}_{10.5}p_{50})\cdot 2+({}_{10.5}p_{50})^2}=\frac{(0.09152)(0.91478)(0.01376)(2)}{(0.09152)(0.90848)(2)+(0.90848)^2}=0.0023$$

where

$${}_{10.5}p_{50}=\frac{\tfrac{1}{2}(l_{60}+l_{61})}{l_{50}}=\frac{\tfrac{1}{2}(8{,}188{,}074+8{,}075{,}403)}{8{,}950{,}901}=0.90848$$

$${}_{10.5}q_{50}=1-{}_{10.5}p_{50}=0.09152$$

$${}_{10}p_{50}=\frac{8{,}188{,}074}{8{,}950{,}901}=0.91478$$

${}_{10.5}p_{50}\;\mu(50+10.5)=({}_{10}p_{50})q_{60}$ since UDD

Alternatively, ${}_{(10+t)}p_{50}={}_{10}p_{50}\;{}_tp_{60}$

$${}_{(10+t)}p_{50:50}=({}_{10}p_{50})^2\,({}_tp_{60})^2$$

$${}_{(10+t)}p_{\overline{50:50}}=2\;{}_{10}p_{50}\;{}_tp_{60}-({}_{10}p_{50})^2\,({}_tp_{60})^2$$

$$=2\;{}_{10}p_{50}(1-tq_{60})-({}_{10}p_{50})^2\,(1-tq_{60})^2\quad\text{since UDD}$$

$$\text{Derivative }=-2\;{}_{10}p_{50}\;q_{60}+2({}_{10}p_{50})^2\,(1-tq_{60})q_{60}$$

Derivative at $10+t=10.5$ is

$$-2(0.91478)(0.01376)+(0.91478)^2\,(1-(0.5)(0.01376))(0.01376)=-0.0023$$

$$ {}_{10.5}p_{\overline{50:50}} = 2\ {}_{10.5}p_{50} - \left({}_{10.5}p_{50}\right)^2 $$
$$ = 2(0.90848) - (0.90848)^2 $$
$$ = 0.99162 $$

$$ \mu \text{ (for any sort of lifetime) } = \frac{-\frac{dp}{dt}}{p} = \frac{-(-0.0023)}{0.99162} = 0.0023 $$

Question #95
Key: D

$$ \mu_x^{(\tau)}(t) = \mu_x^{(1)}(t) + \mu_x^{(2)}(t) = 0.01 + 2.29 = 2.30 $$
$$ P = P\int_0^2 v^t\ {}_tp_x^{(\tau)}\mu_x^{(2)}(t)\ dt + 50,000\int_0^2 v^t\ {}_tp_x^{(\tau)}\mu_x^{(1)}(t)\ dt + 50,000\int_2^\infty v^t\ {}_tp_x^{(\tau)}\mu_x^{(\tau)}(t)\ dt $$
$$ P = P\int_0^2 e^{-0.1t}e^{-2.3t} \times 2.29dt + 50,000\int_0^2 e^{-0.1t}e^{-2.3t} \times 0.01dt + 50,000\int_2^\infty e^{-0.1t}e^{-2.3t} \times 2.3dt $$
$$ P\left[1 - 2.29 \times \frac{1-e^{-2(2.4)}}{2.4}\right] = 50000\left[0.01 \times \frac{1-e^{-2(2.4)}}{2.4} + 2.3 \times \frac{e^{-2(2.4)}}{2.4}\right] $$
P = 11,194

Question #96
Key: B

$$ e_x = p_x + {}_2p_x + {}_3p_x + \ldots = 11.05 $$

$$ \text{Annuity} = v^3\ {}_3p_x 1000 + v^4\ {}_4p_x \times 1000 \times (1.04) + \ldots $$
$$ = \sum_{k=3}^{\infty} 1000(1.04)^{k-3} v^k\ {}_kp_x $$
$$ = 1000v^3 \sum_{k=3}^{\infty} {}_kp_x $$
$$ = 1000v^3(e_x - 0.99 - 0.98) = 1000\left(\frac{1}{1.04}\right)^3 \times 9.08 = 8072 $$

Let π = benefit premium.

$$\pi\left(1+0.99v+0.98v^2\right)=8072$$
$$2.8580\pi=8072$$
$$\pi=2824$$

Question #97
Key B

$$\pi\,\ddot{a}_{30:\overline{10|}}=1000A_{30}+P(IA)^1_{30:\overline{10|}}+(10\pi)\left({}_{10|}A_{30}\right)$$

$$\pi=\frac{1000A_{30}}{\ddot{a}_{30:\overline{10|}}-(IA)^1_{30:\overline{10|}}-10\,{}_{10|}A_{30}}$$
$$=\frac{1000(0.102)}{7.747-0.078-10(0.088)}$$
$$=\frac{102}{6.789}$$
$$=15.024$$

Test Question: 98 Key: E

For de Moivre's law,

$$\mathring{e}_{30}=\int_0^{\omega-30}\left(1-\frac{t}{\omega-30}\right)dt$$
$$=\left[t-\frac{t^2}{2(\omega-30)}\right]_0^{\omega-30}$$
$$=\frac{\omega-30}{2}$$

Prior to medical breakthrough $\omega=100\Rightarrow\mathring{e}_{30}=\dfrac{100-30}{2}=35$

After medical breakthrough $\mathring{e}'_{30}=\mathring{e}_{30}+4=39$

so $\mathring{e}'_{30}=39=\dfrac{\omega'-30}{2}\Rightarrow\omega'=108$

Test Question: 99 Key: A

$_0L = 100{,}000v^{2.5} - 4000\ddot{a}_{\overline{3}|}$ @5%

= 77,079

Question #100
Key: D

$\mu^{(accid)} = 0.001$

$\mu^{(total)} = 0.01$

$\mu^{(other)} = 0.01 - 0.001 = 0.009$

Actuarial present value $= \int_0^\infty 500{,}000\, e^{-0.05t}\, e^{-0.01t}\left(0.009\right)dt$

$$+10\int_0^\infty 50{,}000\, e^{0.04t}\, e^{-0.05t}\, e^{-0.01t}\left(0.001\right)dt$$

$$= 500{,}000\left[\frac{0.009}{0.06} + \frac{0.001}{0.02}\right] = 100{,}000$$

Test Question: 101 Key: E

$E[N] = Var[N] = (60)(0.5) = 30$
$E[X] = (0.6)(1) + (0.2)(5) + (0.2)(10) = 3.6$
$E[X^2] = (0.6)(1) + (0.2)(25) + (0.2)(100) = 25.6$
$Var\ [X] = 25.6 - 3.6^2 = 12.64$

For any compound distribution
$Var\ [S] = E[N]Var[X] + Var[N](E[X])^2$

= (30) (12.64) + (30) (3.6^2)

= 768

For specifically Compound Poisson
$Var[S] = \lambda t\, E[X^2] =$ (60) (0.5) (25.6) = 768

Alternatively, consider this as 3 Compound Poisson processes (coins worth 1; worth 5; worth 10), where for each $Var(X) = 0$, thus for each $Var(S) = Var(N)E[X]^2$.
Processes are independent, so total *Var* is

$$Var = (60)(0.5)(0.6)1^2 + (60)(0.5)(0.2)5^2 + (60)(0.5)(0.2)(10)^2$$
$$= 768$$

Test Question: 102 Key: D

$$1000\ {}_{20}^{20}V_x = 1000A_{x+20} = \frac{1000\left({}_{19}^{20}V_x + {}_{20}P_x\right)(1.06) - q_{x+19}(1000)}{p_{x+19}}$$

$$= \frac{(342.03 + 13.72)(1.06) - 0.01254(1000)}{0.98746} = 369.18$$

$$\ddot{a}_{x+20} = \frac{1 - 0.36918}{(0.06/1.06)} = 11.1445$$

$$so \quad 1000P_{x+20} = 1000\frac{A_{x+20}}{\ddot{a}_{x+20}} = \frac{369.18}{11.1445} = 33.1$$

Test Question: 103 Key: B

$$ {}_k p_x^{(\tau)} = e^{-\int_0^k \mu_x^{(\tau)}(t)dt} = e^{-\int_0^k 2\mu_x^{(1)}(t)dt} $$

$$ = \left(e^{-\int_0^k \mu_x^{(1)}(t)dt} \right)^2 $$

$$ = \left({}_k p_x \right)^2 \text{ where } {}_k p_x \text{ is from Illustrative Life Table, since } \mu^{(1)} \text{ follows I.L.T.} $$

$$ {}_{10} p_{60} = \frac{6{,}616{,}155}{8{,}188{,}074} = 0.80802 $$

$$ {}_{11} p_{60} = \frac{6{,}396{,}609}{8{,}188{,}074} = 0.78121 $$

$$ {}_{10|} q_{60}^{(\tau)} = {}_{10} p_{60}^{(\tau)} - {}_{11} p_{60}^{(\tau)} $$

$$ = \left({}_{10} p_{60} \right)^2 - \left({}_{11} p_{60} \right)^2 \text{ from I.L.T.} $$

$$ = 0.80802^2 - 0.78121^2 = 0.0426 $$

Test Question: 104 Key: C

$P_s = \dfrac{1}{\ddot{a}_s} - d$, where *s* can stand for any of the statuses under consideration.

$$ \ddot{a}_s = \frac{1}{P_s + d} $$

$$ \ddot{a}_x = \ddot{a}_y = \frac{1}{0.1 + 0.06} = 6.25 $$

$$ \ddot{a}_{\overline{xy}} = \frac{1}{0.06 + 0.06} = 8.333 $$

$$ \ddot{a}_{\overline{xy}} + \ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y $$

$$ \ddot{a}_{xy} = 6.25 + 6.25 - 8.333 = 4.167 $$

$$ P_{xy} = \frac{1}{4.167} - 0.06 = 0.18 $$

Test Question: 105 **Key: A**

$$d_0^{(\tau)} = 1000\int_0^1 e^{-(\mu+0.04)t}(\mu+0.04)dt$$
$$= 1000\left(1-e^{-(\mu+0.04)}\right) = 48$$

$$e^{-(\mu+0.04)} = 0.952$$
$$\mu+0.04 = -\ln(0.952)$$
$$= 0.049$$
$$\mu = 0.009$$

$$d_3^{(1)} = 1000\int_3^4 e^{-0.049t}(0.009)dt$$
$$= 1000\frac{0.009}{0.049}\left(e^{-(0.049)(3)} - e^{-(0.049)(4)}\right) = 7.6$$

Question #106
Key: B

This is a graph of $l_x\mu(x)$.
$\mu(x)$ would be increasing in the interval $(80,100)$.
The graphs of $l_x p_x$, l_x and l_x^2 would be decreasing everywhere.

Question #107
Key: B

Variance $= v^{30}\,{}_{15}p_x\,{}_{15}q_x$ Expected value $= v^{15}\,{}_{15}p_x$

$$v^{30}\,{}_{15}p_x\,{}_{15}q_x = 0.065 \quad v^{15}\,{}_{15}p_x$$
$$v^{15}\,{}_{15}q_x = 0.065 \Rightarrow {}_{15}q_x = 0.3157$$

Since μ is constant

$${}_{15}q_x = \left(1-(p_x)^{15}\right)$$
$$(p_x)^{15} = 0.6843$$
$$p_x = 0.975$$
$$q_x = 0.025$$

Question #108
Key: E

$$(1)\quad {}_{11}V^A = \left({}_{10}V^A + 0\right)\frac{(1+i)}{p_{x+10}} - \frac{q_{x+10}}{p_{x+10}} \times 1000$$

$$(2)\quad {}_{11}V^B = \left({}_{10}V^B + \pi^B\right)\frac{(1+i)}{p_{x+10}} - \frac{q_{x+10}}{p_{x+10}} \times 1000$$

$$(1)-(2)\quad {}_{11}V^A - {}_{11}V^B = \left({}_{10}V^A - {}_{10}V^B - \pi^B\right)\frac{(1+i)}{p_{x+10}}$$

$$= (101.35 - 8.36)\frac{(1.06)}{1-0.004}$$

$$= 98.97$$

Test Question: 109 Key: A

A P V (*x*'s benefits) $= \sum_{k=0}^{2} v^{k+1} b_{k+1}\ {}_k p_x\ q_{x+k}$

$$= 1000\left[300v(0.02) + 350v^2(0.98)(0.04) + 400v^3(0.98)(0.96)(0.06)\right]$$

$$= 36{,}829$$

Test Question: 110 Key: E

π denotes benefit premium

${}_{19}V = APV$ future benefits - APV future premiums

$$0.6 = \frac{1}{1.08} - \pi \Rightarrow \pi = 0.326$$

$${}_{11}V = \frac{\left({}_{10}V + \pi\right)(1.08) - (q_{65})(10)}{p_{65}}$$

$$= \frac{(5.0 + 0.326)(1.08) - (0.10)(10)}{1 - 0.10}$$

=5.28

Question #111
Key: A

Actuarial present value Benefits $= \dfrac{(0.8)(0.1)(10,000)}{1.06^2} + \dfrac{(0.8)(0.9)(0.097)(9,000)}{1.06^3}$

$= 1,239.75$

$$1,239.75 = P\left(1 + \frac{(0.8)}{1.06} + \frac{(0.8)(0.9)}{1.06^2}\right)$$

$$= P(2.3955)$$

$$P = 517.53 \quad \Rightarrow 518$$

Test Question: 112 Key: A

$$1180 = 70\bar{a}_{30} + 50\bar{a}_{40} - 20\bar{a}_{30:40}$$

$$1180 = (70)(12) + (50)(10) - 20\bar{a}_{30:40}$$

$$\bar{a}_{30:40} = 8$$

$$\bar{a}_{\overline{30:40}} = \bar{a}_{30} + \bar{a}_{40} - \bar{a}_{30:40} = 12 + 10 - 8 = 14$$

$$100\bar{a}_{\overline{30:40}} = 1400$$

Test Question: 113 Key: B

$$\bar{a} = \int_0^\infty \bar{a}_{\overline{t}|} f(t)dt = \int_0^\infty \frac{1-e^{-0.05t}}{0.05} \frac{1}{\Gamma(2)} te^{-t} dt$$

$$= \frac{1}{0.05}\int_0^\infty \left(te^{-t} - te^{-1.05t}\right)dt$$

$$= \frac{1}{0.05}\left[-(t+1)e^{-t} + \left(\frac{t}{1.05} + \frac{1}{1.05^2}\right)e^{-1.05t}\right]\Bigg|_0^\infty$$

$$= \frac{1}{0.05}\left[1 - \left(\frac{1}{1.05}\right)^2\right] = 1.85941$$

$20{,}000 \times 1.85941 = 37{,}188$

Question #114
Key: C

<u>Event</u>	<u>***Prob***</u>	<u>Present Value</u>
$x = 0$	(0.05)	15
$x = 1$	$(0.95)(0.10) = 0.095$	$15 + 20/1.06 = 33.87$
$x \geq 2$	$(0.95)(0.90) = 0.855$	$15 + 20/1.06 + 25/1.06^2 = 56.12$

$E[X] = (0.05)(15) + (0.095)(33.87) + (0.855)(56.12) = 51.95$

$E\left[X^2\right] = (0.05)(15)^2 + (0.095)(33.87)^2 + (0.855)(56.12)^2 = 2813.01$

$Var[X] = E\left(X^2\right) - E(X)^2 = 2813.01 - (51.95)^2 = 114.2$

Question #115
Key: B

Let *K* be the curtate future lifetime of (*x* + *k*)

${}_kL = 1000v^{K+1} - 1000P_{x:\overline{3}|} \times \ddot{a}_{\overline{K+1}|}$

When (as given in the problem), (*x*) dies in the second year from issue, the curtate future lifetime of $(x+1)$ is 0, so

$$ {}_1L = 1000v - 1000\,P_{x:\overline{3}|}\,\ddot{a}_{\overline{1}|} $$
$$ = \frac{1000}{1.1} - 279.21 $$
$$ = 629.88 \approx 630 $$

The premium came from

$$ P_{x:\overline{3}|} = \frac{A_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}} $$
$$ A_{x:\overline{3}|} = 1 - d\,\ddot{a}_{x:\overline{3}|} $$
$$ P_{x:\overline{3}|} = 279.21 = \frac{1 - d\,\ddot{a}_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}} = \frac{1}{\ddot{a}_{x:\overline{3}|}} - d $$

Test Question: 116 Key: D

Let M = the force of mortality of an individual drawn at random; and $T =$ future lifetime of the individual.

$$ \Pr\left[T \le 1\right] = E\left\{\Pr\left[T \le 1 \middle| M\right]\right\} $$
$$ = \int_0^\infty \Pr\left[T \le 1 \middle| M = \mu\right] f_M(\mu)\,d\mu $$
$$ = \int_0^2 \int_0^1 \mu e^{-\mu t}\,dt\,\frac{1}{2}\,d\mu $$
$$ = \int_0^2 \left(1 - e^{-\mu}\right)\frac{1}{2}\,du = \frac{1}{2}\left(2 + e^{-2} - 1\right) = \frac{1}{2}\left(1 + e^{-2}\right) $$
$$ = 0.56767 $$

Question #117
Key: E

Note that above 40, decrement 1 is DeMoivre with omega = 100; decrement 2 is DeMoivre with omega = 80.

That means $\mu_{40}^{(1)}(20) = 1/40 = 0.025;\ \mu_{40}^{(2)}(20) = 1/20 = 0.05$

$$ \mu_{40}^{(\tau)}(20) = 0.025 + 0.05 = 0.075 $$

Or from basic definition of μ ,

$$ {}_t p_{40}^{(\tau)} = \frac{60-t}{60} \times \frac{40-t}{40} = \frac{2400 - 100t + t^2}{2400} $$

$d\left({}_t p_{40}^{(\tau)}\right)/dt = (-100+2t)/2400$

at $t=20$ gives $-60/2400 = 0.025$

${}_{20}p_{40}^{(\tau)} = (2/3)*(1/2) = 1/3$

$\mu_{40}^{(\tau)}(20) = \left[-d\left({}_t p_{40}^{(\tau)}\right)/dt\right]/\ {}_{20}p_{40}^{(\tau)} = 0.025/(1/3) = 0.075$

Test Question: 118 Key: D

Let $\pi =$ benefit premium

Actuarial present value of benefits =

$$= (0.03)(200{,}000)v + (0.97)(0.06)(150{,}000)v^2 + (0.97)(0.94)(0.09)(100{,}000)v^3$$
$$= 5660.38 + 7769.67 + 6890.08$$
$$= 20{,}320.13$$

Actuarial present value of benefit premiums

$$= \ddot{a}_{x:\overline{3}|}\,\pi$$
$$= \left[1 + 0.97v + (0.97)(0.94)v^2\right]\pi$$
$$= 2.7266\,\pi$$
$$\pi = \frac{20{,}320.13}{2.7266} = 7452.55$$

$${}_1V = \frac{(7452.55)(1.06) - (200{,}000)(0.03)}{1-0.03}$$
$$= 1958.46$$

Initial reserve, year 2 = ${}_1V + \pi$

= 1958.56 + 7452.55

= 9411.01

Test Question: 119 Key: A

Let π denote the premium.

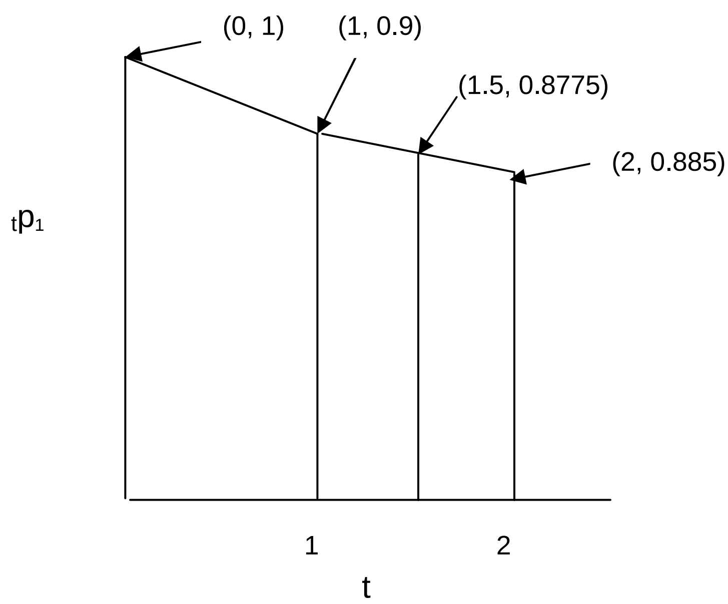
$$L = b_T v^T - \pi\,\bar{a}_{\overline{T}|} = (1+i)^T \times v^T - \pi\,\bar{a}_{\overline{T}|}$$
$$= 1 - \pi\,\bar{a}_{\overline{T}|}$$

$$E[L] = 1 - \pi\,\bar{a}_x = 0 \quad \Rightarrow \pi = 1/\bar{a}_x$$

$$\Rightarrow L = 1 - \pi \bar{a}_{\overline{T|}} = 1 - \frac{\bar{a}_{\overline{T|}}}{\bar{a}_x} = \frac{\delta \bar{a}_x - \left(1 - v^T\right)}{\delta \bar{a}_x}$$

$$= \frac{v^T - \left(1 - \delta \bar{a}_x\right)}{\delta \bar{a}_x} = \frac{v^T - \bar{A}_x}{1 - \bar{A}_x}$$

Test Question: 120 Key: D



$${}_1 p_1 = (1 - 0.1) = 0.9$$
$${}_2 p_1 = (0.9)(1 - 0.05) = 0.855$$

since uniform, ${}_{1.5}p_1 = \left(0.9 + 0.855\right)/2$

$$= 0.8775$$

$\overset{\circ}{e}_{1:\overline{1.5|}} =$ Area between $t = 0$ and $t = 1.5$

$$= \left(\frac{1 + 0.9}{2}\right)(1) + \left(\frac{0.9 + 0.8775}{2}\right)(0.5)$$
$$= 0.95 + 0.444$$
$$= 1.394$$

Alternatively,

$$\overset{\circ}{e}_{1:\overline{1.5|}} = \int_0^{1.5} {}_t p_1 dt$$

$$= \int_0^1 {}_t p_1 dt + {}_1 p_1 \int_0^{0.5} {}_x p_2 dx$$

$$= \int_0^1 (1 - 0.1t) dt + 0.9 \int_0^{0.5} (1 - 0.05x) dx$$

$$= \left[t - \tfrac{0.1t^2}{2} \right]_0^1 + 0.9 \left[x - \tfrac{0.05x^2}{2} \right]_0^{0.5}$$

$$= 0.95 + 0.444 = 1.394$$

Test Question: 121 Key: A

$$10{,}000 A_{63} (1.12) = 5233$$

$$A_{63} = 0.4672$$

$$A_{x+1} = \frac{A_x (1+i) - q_x}{p_x}$$

$$A_{64} = \frac{(0.4672)(1.05) - 0.01788}{1 - 0.01788}$$

$$= 0.4813$$

$$A_{65} = \frac{(0.4813)(1.05) - 0.01952}{1 - 0.01952}$$

$$= 0.4955$$

Single contract premium at 65 = (1.12) (10,000) (0.4955)
= 5550

$$(1+i)^2 = \frac{5550}{5233} \qquad i = \sqrt{\frac{5550}{5233}} - 1 = 0.02984$$

Test Question: 122 Key: B

Original Calculation (assuming independence):

$$\mu_x = 0.06$$
$$\mu_y = 0.06$$
$$\mu_{xy} = 0.06 + 0.06 = 0.12$$
$$\bar{A}_x = \frac{\mu_x}{\mu_x + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_y = \frac{\mu_y}{\mu_y + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.12}{0.12 + 0.05} = 0.70588$$
$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.70588 = 0.38502$$

Revised Calculation (common shock model):

$$\mu_x = 0.06,\ \mu_x^{T^*(x)} = 0.04$$
$$\mu_y = 0.06,\ \mu_y^{T^*(y)} = 0.04$$
$$\mu_{xy} = \mu_x^{T^*(x)} + \mu_y^{T^*(y)} + \mu^Z + 0.04 + 0.04 + 0.02 = 0.10$$
$$\bar{A}_x = \frac{\mu_x}{\mu_x + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_y = \frac{\mu_y}{\mu_{y+\delta}} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.10}{0.10 + 0.05} = 0.66667$$
$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Difference $= 0.42423 - 0.38502 = 0.03921$

Question #123
Key: B

$$
\begin{aligned}
{}_{5|}q_{\overline{35:45}} &= {}_{5|}q_{35} + {}_{5|}q_{45} - {}_{5|}q_{35:45} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35:45}q_{40:50} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45}\left(1 - p_{40:50}\right) \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45}\left(1 - p_{40}p_{50}\right) \\
&= (0.9)(.03) + (0.8)(0.05) - (0.9)(0.8)\left[1 - (0.97)(0.95)\right] \\
&= 0.01048
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
{}_6p_{35} &= {}_5p_{35} \times p_{40} = (0.90)(1 - 0.03) = 0.873 \\
{}_6p_{45} &= {}_5p_{45} \times p_{50} = (0.80)(1 - 0.05) = 0.76
\end{aligned}
$$

$$
\begin{aligned}
{}_{5|}q_{\overline{35:45}} &= {}_5p_{\overline{35:45}} - {}_6p_{\overline{35:45}} \\
&= \left({}_5p_{35} + {}_5p_{45} - {}_5p_{35:45}\right) - \left({}_6p_{35} + {}_6p_{45} - {}_6p_{35:45}\right) \\
&= \left({}_5p_{35} + {}_5p_{45} + {}_5p_{35} \times {}_5p_{45}\right) - \left({}_6p_{35} + {}_6p_{45} - {}_6p_{35} \times {}_6p_{45}\right) \\
&= (0.90 + 0.80 - 0.90 \times 0.80) - (0.873 + 0.76 - 0.873 \times 0.76) \\
&= 0.98 - 0.96952 \\
&= 0.01048
\end{aligned}
$$

Test Question: 124 Key: C

$\int_0^3 \lambda(t)dt = 6$ so $N(3)$ is Poisson with $\lambda = 6.$

P is Poisson with mean 3 (with mean 3 since $\text{Prob}(y_i < 500) = 0.5$)

P and *Q* are independent, so the mean of P is 3, no matter what the value of *Q* is.

Test Question: 125 Key: A

At age *x*:

Actuarial Present value (APV) of future benefits = $\left(\frac{1}{5}A_x\right)1000$

APV of future premiums = $\left(\frac{4}{5}\ddot{a}_x\right)\pi$

$\frac{1000}{5}A_{25} = \frac{4}{5}\pi\,\ddot{a}_{25}$ by equivalence principle

$\frac{1000}{4}\frac{A_{25}}{\ddot{a}_{25}} = \pi \Rightarrow \pi = \frac{1}{4}\times\frac{81.65}{16.2242} = 1.258$

${}_{10}V =$ APV (Future benefits) – APV (Future benefit premiums)

$= \frac{1000}{5}A_{35} - \frac{4}{5}\pi\,\ddot{a}_{35}$

$= \frac{1}{5}(128.72) - \frac{4}{5}(1.258)(15.3926)$

$= 10.25$

Test Question: 126 Key: E

Let Y = present value random variable for payments on one life

$S = \sum Y =$ present value random variable for all payments

$E[Y] = 10\ddot{a}_{40} = 148.166$

$\text{Var}[Y] = 10^2\,\frac{\left({}^2A_{40} - A_{40}^2\right)}{d^2}$

$= 100\left(0.04863 - 0.16132^2\right)(1.06/0.06)^2$

$= 705.55$

$E[S] = 100E[Y] = 14{,}816.6$

$\text{Var}[S] = 100\,\text{Var}[Y] = 70{,}555$

Standard deviation $[S] = \sqrt{70{,}555} = 265.62$

By normal approximation, need

E [S] + 1.645 Standard deviations = 14,816.6 + (1.645) (265.62)

= 15,254

Test Question: 127 Key: B

Initial Benefit Prem $= \dfrac{5A_{30} - 4\left(A^{\;1}_{30:\overline{20}|}\right)}{5\ddot{a}_{30:\overline{35}|} - 4\ddot{a}_{30:\overline{20}|}}$

$$= \frac{5(0.10248) - 4(0.02933)}{5(14.835) - 4(11.959)}$$

$$= \frac{0.5124 - 0.11732}{74.175 - 47.836} = \frac{0.39508}{26.339} = 0.015$$

Where

$A^{\;1}_{30:\overline{20}|} = \left(A_{30:\overline{20}|} - A_{30:\overline{20}|}^{\;\;\;\;\;\;\;1}\right) = 0.32307 - 0.29374 = 0.02933$

and

$\ddot{a}_{30:\overline{20}|} = \dfrac{1 - A_{30:\overline{20}|}}{d} = \dfrac{1 - 0.32307}{\left(\dfrac{0.06}{1.06}\right)} = 11.959$

Comment: the numerator could equally well have been calculated as $A_{30} + 4\ {}_{20}E_{30}\ A_{50}$
= 0.10248 + (4) (0.29374) (0.24905)
= 0.39510

Test Question: 128 Key: B

$${}_{0.75}p_x = 1 - (0.75)(0.05)$$
$$= 0.9625$$
$${}_{0.75}p_y = 1 - (0.75)(.10)$$
$$= 0.925$$
$${}_{0.75}q_{xy} = 1 - {}_{0.75}p_{xy}$$
$$= 1 - \left({}_{0.75}p_x\right)\left({}_{0.75}p_y\right) \text{ since independent}$$
$$= 1 - (0.9625)(0.925)$$
$$= 0.1097$$

Question #129
Key: D

Let *G* be the expense-loaded premium.
Actuarial present value (APV) of benefits = $100{,}000A_{35}$
APV of premiums = $G\ddot{a}_{35}$
APV of expenses = $\left[0.1G + 25 + (2.50)(100)\right]\ddot{a}_{35}$
Equivalence principle:

$$G\ddot{a}_{35} = 100,000A_{35} + \left(0.1G + 25 + 250\right)\ddot{a}_{35}$$

$$G = 100,000\frac{A_{35}}{\ddot{a}_{35}} + 0.1G + 275$$

$$0.9G = 100,000P_{35} + 275$$

$$G = \frac{(100)(8.36) + 275}{0.9}$$

$$= 1234$$

Test Question: 130 Key: A

The person receives *K* per year guaranteed for 10 years $\Rightarrow K\ddot{a}_{\overline{10}|} = 8.4353K$

The person receives *K* per years alive starting 10 years from now $\Rightarrow {}_{10|}\ddot{a}_{40}K$

*Hence we have $10000 = \left(8.4353 + {}_{10}E_{40}\ddot{a}_{50}\right)K$

Derive ${}_{10}E_{40}$:

$$A_{40} = A^{1}_{40:\overline{10}|} + \left({}_{10}E_{40}\right)A_{50}$$

$${}_{10}E_{40} = \frac{A_{40} - A^{1}_{40:\overline{10}|}}{A_{50}} = \frac{0.30 - 0.09}{0.35} = 0.60$$

Derive $\ddot{a}_{50} = \frac{1 - A_{50}}{d} = \frac{1 - 0.35}{\frac{.04}{1.04}} = 16.90$

Plug in values:

$$10,000 = \left(8.4353 + (0.60)(16.90)\right)K$$

$$= 18.5753K$$

$$K = 538.35$$

Test Question: 131 Key: D

STANDARD: $\mathring{e}_{25:\overline{11}|} = \int_0^{11}\left(1 - \frac{t}{75}\right)dt = t - \frac{t^2}{2 \times 75}\Big|_0^{11} = 10.1933$

MODIFIED: $p_{25} = e^{-\int_0^1 0.1ds} = e^{-.1} = 0.90484$

$$\mathring{e}_{25:\overline{11}|} = \int_0^1 {}_t p_{25}\,dt + p_{25}\int_0^{10}\left(1 - \frac{t}{74}\right)dt$$

$$= \int_0^1 e^{-0.1t} dt + e^{-0.1} \int_0^{10} \left(1 - \frac{t}{74}\right) dt$$

$$= \frac{1 - e^{-0.1}}{0.1} + e^{-0.1} \left(t - \frac{t^2}{2 \times 74} \right) \Big|_0^{10}$$

$$= 0.95163 + 0.90484(9.32432) = 9.3886$$

Difference =0.8047

Test Question: 132 Key: B

Comparing B & D: Prospectively at time 2, they have the same future benefits. At issue, B has the lower benefit premium. Thus, by formula 7.2.2, B has the higher reserve.

Comparing A to B: use formula 7.3.5. At issue, B has the higher benefit premium. Until time 2, they have had the same benefits, so B has the higher reserve.

Comparing B to C: Visualize a graph C* that matches graph B on one side of t=2 and matches graph C on the other side. By using the logic of the two preceding paragraphs, C's reserve is lower than C*'s which is lower than B's.

Comparing B to E: Reserves on E are constant at 0.

Test Question: 133 Key: C

Since only decrements (1) and (2) occur during the year, probability of reaching the end of the year is

$$p'^{(1)}_{60} \times p'^{(2)}_{60} = (1 - 0.01)(1 - 0.05) = 0.9405$$

Probability of remaining through the year is

$$p'^{(1)}_{60} \times p'^{(2)}_{60} \times p'^{(3)}_{60} = (1 - 0.01)(1 - 0.05)(1 - 0.10) = 0.84645$$

Probability of exiting at the end of the year is

$$q^{(3)}_{60} = 0.9405 - 0.84645 = 0.09405$$

Question #134
Key: D

Poisoned wine glasses are drunk at a Poisson rate of 2×0.01 = 0.02 per day.
Number of glasses in 30 days is Poisson with $\lambda = 0.02 \times 30 = 0.60$

$f(0) = e^{-0.60} = 0.55$

Test Question: 135 Key: D

APV of regular death benefit $= \int_0^\infty (100000)\left(e^{-\delta t}\right)(0.008)\left(e^{-\mu t}\right)dt$

$= \int_0^\infty (100000)\left(e^{-0.06t}\right)(0.008)\left(e^{-0.008t}\right)dt$

$= 100000\left[0.008 / (0.06 + 0.008)\right] = 11{,}764.71$

APV of accidental death benefit $= \int_0^{30} (100000)\left(e^{-\delta t}\right)(0.001)\left(e^{-\mu t}\right)dt$

$= \int_0^{30} (100000)\left(e^{-0.06t}\right)(0.001)\left(e^{-0.008t}\right)dt$

$= 100\left[1 - e^{-2.04}\right] / 0.068 = 1{,}279.37$

Total APV $= 11765 + 1279 = 13044$

Test Question: 136 Key: B

$$l_{[60]+.6} = (.6)(79{,}954) + (.4)(80{,}625)$$
$$= 80{,}222.4$$
$$l_{[60]+1.5} = (.5)(79{,}954) + (.5)(78{,}839)$$
$$= 79{,}396.5$$

$$_{0.9}q_{[60]+.6} = \frac{80222.4 - 79{,}396.5}{80{,}222.4}$$
$$= 0.0103$$

$$P_0 = \tfrac{1}{11} = 9.09\overline{09}\%$$

Question #137
Key: E

View the compound Poisson process as two compound Poisson processes, one for smokers and one for non-smokers. These processes are independent, so the total variance is the sum of their variances.

For smokers, $\lambda = (0.2)(1000) = 200$

Var(losses) $= \lambda\left[\text{Var}(X) + \left(E(X)\right)^2\right]$

$= 200\left[5000 + (-100)^2\right]$

$= 3{,}000{,}000$

For non-smokers, $\lambda = (0.8)(1000) = 800$

Var(losses) $= \lambda\left[\text{Var}(X) + \left(E(X)\right)^2\right]$

$= 800\left[8000 + (-100)^2\right]$

$= 14{,}400{,}000$

Total variance $=$ 3,000,000 + 14,400,000
$=$ 17,400,000

Test Question: 138 Key: A

$$q_{40}^{(\tau)} = q_{40}^{(1)} + q_{40}^{(2)} = 0.34$$
$$= 1 - p_{40}'^{(1)} p_{40}'^{(2)}$$
$$0.34 = 1 - 0.75 p_{40}'^{(2)}$$

$$p_{40}'^{(2)} = 0.88$$
$$q_{40}'^{(2)} = 0.12 = y$$

$$q_{41}'^{(2)} = 2y = 0.24$$
$$q_{41}^{(\tau)} = 1 - (0.8)(1 - 0.24) = 0.392$$
$$l_{42}^{(\tau)} = 2000(1 - 0.34)(1 - 0.392) = 803$$

Test Question: 139 Key: C

$\Pr\left[L(\pi')>0\right]<0.5$

$\Pr\left[10{,}000v^{K+1}-\pi'\ddot{a}_{\overline{K+1|}}>0\right]<0.5$

From Illustrative Life Table, ${}_{47}p_{30}=0.50816$ and ${}_{48}p_{30}=.47681$

Since *L* is a decreasing function of *K*, to have
$\Pr\left[L(\pi')>0\right]<0.5$ means we must have $L(\pi')\le 0$ for $K\ge 47$.
Highest value of $L(\pi')$ for $K\ge 47$ is at *K* = 47.

$L(\pi')\left[\text{at } K=47\right]=10{,}000\,v^{47+1}-\pi'\,\ddot{a}_{\overline{47+1|}}$

$=609.98-16.589\pi'$

$L(\pi')\le 0\Rightarrow\left(609.98-16.589\,\pi'\right)\le 0$

$$\Rightarrow\pi'>\frac{609.98}{16.589}=36.77$$

Test Question: 140 Key: B

$\Pr(K=0)=1-p_x=0.1$

$\Pr(K=1)={}_1p_x-{}_2p_x=0.9-0.81=0.09$

$\Pr(K>1)={}_2p_x=0.81$

$E(Y)=.1\times 1+.09\times 1.87+.81\times 2.72=2.4715$

$E\left(Y^2\right)=.1\times 1^2+.09\times 1.87^2+.81\times 2.72^2=6.407$

$$VAR(Y)=6.407-2.4715^2=0.299$$

Question #141
Key: E

$E[Z]=b\bar{A}_x$

since constant force $\bar{A}_x=\mu/\mu+\delta$

$$E(Z)=\frac{b\mu}{\mu+\delta}=\frac{b(0.02)}{(0.06)}=b/3$$

$$\mathrm{Var}[Z] = \mathrm{Var}\left[b v^{T}\right] = b^2 \mathrm{Var}\left[v^{T}\right] = b^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$= b^2\left(\frac{\mu}{\mu+2\delta} - \left(\frac{\mu}{\mu+\delta}\right)^2\right)$$

$$= b^2\left[\frac{2}{10} - \frac{1}{9}\right] = b^2\left(\frac{4}{45}\right)$$

$$\mathrm{Var}(Z) = E(Z)$$

$$b^2\left[\frac{4}{45}\right] = \frac{b}{3}$$

$$b\left[\frac{4}{45}\right] = \frac{1}{3} \Rightarrow b = 3.75$$

Test Question: 142 Key: B

In general $\mathrm{Var}(L) = \left(1+\frac{P}{\delta}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$

Here $\bar{P}(\bar{A}_x) = \frac{1}{\bar{a}_x} - \delta = \frac{1}{5} - .08 = .12$

So $\mathrm{Var}(L) = \left(1+\frac{.12}{.08}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right) = .5625$

and $\mathrm{Var}(L^*) = \left(1+\frac{\frac{5}{4}(.12)}{.08}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$

So $\mathrm{Var}(L^*) = \frac{\left(1+\frac{15}{8}\right)^2}{\left(1+\frac{12}{8}\right)^2}(0.5625) = .744$

$\mathrm{E}[L^*] = \bar{A}_x - .15\bar{a}_x = 1 - \bar{a}_x(\delta + .15) = 1 - 5(.23) = -.15$

$\mathrm{E}[L^*] + \sqrt{\mathrm{Var}(L^*)} = .7125$

Test Question: 143 Key: C

Serious claims are reported according to a Poisson process at an average rate of 2 per month. The chance of seeing at least 3 claims is (1 – the chance of seeing 0, 1, or 2 claims).

$P(3+) \geq 0.9$ is the same as $P(0,1,2) \leq 0.1$ is the same as $\left[P(0) + P(1) + P(2)\right] \leq 0.1$

$0.1 \geq e^{-\lambda} + \lambda e^{-\lambda} + \left(\lambda^2 / 2\right)e^{-\lambda}$

The expected value is 2 per month, so we would expect it to be at least 2 months $\left(\lambda = 4\right)$.
Plug in and try

$e^{-4} + 4e^{-4} + \left(4^2 / 2\right)e^{-4} = .238,$ too high, so try 3 months $\left(\lambda = 6\right)$

$e^{-6} + 6e^{-6} + \left(6^2 / 2\right)e^{-6} = .062,$ okay. The answer is 3 months.

[While 2 is a reasonable first guess, it was not critical to the solution. Wherever you start, you should conclude 2 is too few, and 3 is enough].

Test Question: 144 Key: B

Let $l_0^{(\tau)} =$ number of students entering year 1
superscript (*f*) denote academic failure
superscript (*w*) denote withdrawal
subscript is "age" at start of year; equals year - 1

$p_0^{(\tau)} = 1 - 0.40 - 0.20 = 0.40$

$l_2^{(\tau)} = 10\, l_2^{(\tau)}\, q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$

$$q_2^{(w)} = q_2^{(\tau)} - q_2^{(f)} = \left(1.0 - 0.6\right) - 0.1 = 0.3$$

$$l_1^{(\tau)} q_1^{(f)} = 0.4\left[l_1^{(\tau)}\left(1 - q_1^{(f)} - q_1^{(w)}\right)\right]$$

$$q_1^{(f)} = 0.4\left(1 - q_1^{(f)} - 0.3\right)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(\tau)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$$_3q_0^{(w)} = q_0^{(w)} + p_0^{(\tau)} q_1^{(w)} + p_0^{(\tau)} p_1^{(\tau)} q_2^{(w)}$$

$$= 0.2 + \left(0.4\right)\left(0.3\right) + \left(0.4\right)\left(0.5\right)\left(0.3\right)$$

$$= 0.38$$

Test Question: 145 Key: D

$$e_{25} = p_{25}(1+e_{26})$$

$$e_{26}^{N} = e_{26}^{M} \quad \text{since same } \mu$$

$$p_{25}^{N} = e^{-\int_0^1 \left[\mu_{25}^{M}(t)+0.1(1-t)\right]dt}$$

$$= e^{-\int_0^1 \mu_{25}^{M}(t)dt - \int_0^1 0.1(1-t)dt}$$

$$= e^{-\int_0^1 \mu_{25}^{M}(t)dt} e^{-\int_0^1 0.1(1-t)dt}$$

$$= p_{25}^{M} e^{-\left[0.1\left(t-\frac{t^2}{2}\right)\right]_0^1}$$

$$= e^{-0.05} p_{25}^{M}$$

$$e_{25}^{N} = p_{25}^{N}(1+e_{26})$$

$$= e^{-0.05} p_{25}^{M}(1+e_{26})$$

$$= 0.951\, e_{25}^{M} = (0.951)(10.0) = 9.5$$

Test Question: 146 Key: D

$$\mathrm{E}\left[Y_{AGG}\right] = 100\mathrm{E}\left[Y\right] = 100(10{,}000)\bar{a}_x$$

$$= 100(10{,}000)\left(\frac{\left(1-\bar{A}_x\right)}{\delta}\right) = 10{,}000{,}000$$

$$\sigma_Y = \sqrt{\mathrm{Var}\left[Y\right]} = \sqrt{(10{,}000)^2 \frac{1}{\delta^2}\left({}^2\bar{A}_x - \bar{A}_x^2\right)}$$

$$= \frac{(10{,}000)}{\delta}\sqrt{(0.25)-(0.16)} = 50{,}000$$

$$\sigma_{AGG} = \sqrt{100}\sigma_Y = 10(50{,}000) = 500{,}000$$

$$0.90 = \Pr\left[\frac{F - \mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}} > 0\right]$$

$$\Rightarrow 1.282 = \frac{F - \mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}}$$

$$F = 1.282\sigma_{AGG} + \mathrm{E}\left[Y_{AGG}\right]$$

$$F = 1.282(500{,}000) + 10{,}000{,}000 = 10{,}641{,}000$$

Question #147
Key: A

$$A^1_{30:\overline{3}|} = 1000vq_{30} + 500v^2\,{}_{1|}q_{30} + 250v^3\,{}_{2|}q_{30}$$

$$= 1000\left(\frac{1}{1.06}\right)\left(\frac{1.53}{1000}\right) + 500\left(\frac{1}{1.06}\right)^2(0.99847)\left(\frac{1.61}{1000}\right) + 250\left(\frac{1}{1.06}\right)^3(0.99847)(0.99839)\left(\frac{1.70}{1000}\right)$$

$$= 1.4434 + 0.71535 + 0.35572 = 2.51447$$

$$\ddot{a}^{(2)}_{30:\overline{1}|} = \tfrac{1}{2} + \tfrac{1}{2}\left(\frac{1}{1.06}\right)^{1/2}\left(1 - \tfrac{1}{2}q_{30}\right) = \frac{1}{2} + \frac{1}{2}(0.97129)\left(1 - \frac{0.00153}{2}\right)$$

$$= \frac{1}{2} + \frac{1}{2}(0.97129)(0.999235)$$

$$= 0.985273$$

Annualized premium $= \dfrac{2.51447}{0.985273}$

$= 2.552$

Each semiannual premium $= \dfrac{2.552}{2}$

$= 1.28$

Test Question: 148 Key: E

$$(DA)^{1}_{80:\overline{20}|} = 20vq_{80} + vp_{80}\left((DA)^{1}_{81:\overline{19}|}\right)$$

$q_{80} = .2$ $\quad 13 = \frac{20(.2)}{1.06} + \frac{.8}{1.06}(DA)^{1}_{81:\overline{19}|}$

$$\therefore (DA)^{1}_{81:\overline{19}|} = \frac{13(1.06)-4}{.8} = 12.225$$

$q_{80} = .1$ $\quad DA^{1}_{80:\overline{20}|} = 20v(.1) + v(.9)(12.225)$

$$= \frac{2+.9(12.225)}{1.06} = 12.267$$

Test Question: 149 Key: B

Let T denote the random variable of time until the college graduate finds a job
Let $\{N(t), t \geq 0\}$ denote the job offer process

Each offer can be classified as either

$$\begin{cases} \text{Type I - - accept with probability } p \Rightarrow \{N_1(t)\} \\ \text{Type II - - reject with probability } (1-p) \Rightarrow \{N_2(t)\} \end{cases}$$

$\{N_1(t)\}$ is Poisson process with $\lambda_1 = \lambda \cdot p$

$$p = \Pr(w > 28{,}000) = \Pr(\ln w > \ln 28{,}000)$$

$$= \Pr(\ln w > 10.24) = \Pr\left(\frac{\ln w - 10.12}{0.12} > \frac{10.24 - 10.12}{0.12}\right) = 1 - \Phi(1)$$

$$= 0.1587$$

$$\lambda_1 = 0.1587 \times 2 = 0.3174$$

T has an exponential distribution with $\theta = \frac{1}{.3174} = 3.15$

$$\Pr(T > 3) = 1 - F(3)$$

$$= e^{\frac{-3}{3.15}} = 0.386$$

Test Question: 150 Key: A

$$ {}_t p_x = \exp\left[-\int_0^t \frac{ds}{100-x-s}\right] = \exp\left[\ln(100-x-s)\Big|_0^t\right] = \frac{100-x-t}{100-x} $$

$$ \mathring{e}_{\overline{50:60}} = \mathring{e}_{50} + \mathring{e}_{60} - \mathring{e}_{50:60} $$

$$ \mathring{e}_{50} = \int_0^{50} \frac{50-t}{50}\,dt = \frac{1}{50}\left[50t - \frac{t^2}{2}\right]_0^{50} = 25 $$

$$ \mathring{e}_{60} = \int_0^{40} \frac{40-t}{40}\,dt = \frac{1}{40}\left[40t - \frac{t^2}{2}\right]_0^{40} = 20 $$

$$ \mathring{e}_{50:60} = \int_0^{40}\left(\frac{50-t}{50}\right)\left(\frac{40-t}{40}\right)dt = \int_0^{40}\frac{1}{2000}\left(2000-90t+t^2\right)dt $$

$$ = \frac{1}{2000}\left(2000t - 45t^2 + \frac{t^3}{3}\Big|_0^{40}\right) = 14.67 $$

$$ \mathring{e}_{\overline{50:60}} = 25 + 20 - 14.67 = 30.33 $$

Question #151
Key: C

Ways to go $0 \to 2$ in 2 years

$$ 0-0-2;\ p = (0.7)(0.1) = 0.07 $$

$$ 0-1-2;\ p = (0.2)(0.25) = 0.05 $$

$$ 0-2-2;\ p = (0.1)(1) = 0.1 $$

Total = 0.22
Binomial m = 100 q = 0.22
Var = (100) (0.22) (0.78) = 17

Question #152
Key: A

For death occurring in year 2

$$ APV = \frac{0.3 \times 1000}{1.05} = 285.71 $$

For death occurring in year 3, two cases:

(1) State 2 $\rightarrow$ State 1 $\rightarrow$ State 4: (0.2×0.1) = 0.02
(2) State 2 $\rightarrow$ State 2 $\rightarrow$ State 4: (0.5×0.3) = 0.15
Total 0.17

APV = $\dfrac{0.17 \times 1000}{1.05^2} = 154.20$

Total. APV = 285.71 + 154.20 = 439.91

Test Question: 153 Key: E

$$\mathrm{Var}({}_0L) = \mathrm{Var}(\Lambda_0) + v^2 \mathrm{Var}(\Lambda_1) \quad \text{since } Var(\Lambda_2) = 0$$

$$\mathrm{Var}(\Lambda_0) = \left[v(b_1 - {}_1V)\right]^2 p_{50} q_{50}$$
$$= \frac{(10{,}000 - 3{,}209)^2 (0.00832)(0.99168)}{1.03^2}$$
$$= 358664.09$$

$$\mathrm{Var}(\Lambda_1) = \left[v(b_2 - {}_2V)\right]^2 p_{50} q_{51} p_{51}$$
$$= \frac{(10{,}000 - 6{,}539)^2 (0.99168)(0.00911)(0.99089)}{1.03^2}$$
$$= 101075.09$$

$$\mathrm{Var}({}_0L) = 358664.09 + \frac{101075.09}{1.03^2} = 453937.06$$

Alternative solution:

$$\pi = 10{,}000\, v - {}_2V_{50:\overline{3}|} = 9708.74 - 6539 = 3169.74$$

$$ {}_0L = \begin{cases} 10{,}000\, v - \pi\, \ddot{a}_{\overline{1}|} = 6539 & \text{for} \quad K = 0 \\ 10{,}000\, v^2 - \pi\, \ddot{a}_{\overline{2}|} = 3178.80 & \text{for} \quad K = 1 \\ 10{,}000\, v^3 - \pi\, \ddot{a}_{\overline{3}|} = -83.52 & \text{for} \quad K > 1 \end{cases} $$

$\Pr(K=0) = q_{50} = 0.00832$

$\Pr(K=1) = p_{50}\, q_{51} = (0.99168)(0.00911) = 0.0090342$

$\Pr(K>1) = 1 - \Pr(K=0) - \Pr(K=1) = 0.98265$

$\mathrm{Var}({}_0L) = \mathrm{E}\left[{}_0L^2\right] - \mathrm{E}\left[{}_0L\right]^2 = \mathrm{E}\left[{}_0L^2\right]$ since π is benefit premium

$= 0.00832 \times 6539^2 + 0.00903 \times 3178.80^2 + 0.98265 \times (-83.52)^2$

$= 453{,}895$ [difference from the other solution is due to rounding]

Test Question: 154 Key: C

Let π denote the single benefit premium.

$\pi = {}_{30|}\ddot{a}_{35} + \pi\, A^{1}_{35:\overline{30}|}$

$$\pi = \frac{{}_{30|}\ddot{a}_{35}}{1 - A^{1}_{35:\overline{30}|}} = \frac{\left(A_{35:\overline{30}|} - A^{1}_{35:\overline{30}|}\right)\ddot{a}_{65}}{1 - A^{1}_{35:\overline{30}|}} =$$

$$= \frac{(.21-.07)9.9}{(1-.07)}$$

$$= \frac{1.386}{.93}$$

$$= 1.49$$

Test Question: 155 Key: E

$${}_{0.4}p_0 = .5 = e^{-\int_0^{0.4}\left(F + e^{2x}\right)dx}$$

$$= e^{-.4F - \left[\frac{e^{2x}}{2}\right]_0^{.4}}$$

$$= e^{-.4F - \left(\frac{e^{0.8}-1}{2}\right)}$$

$$.5 = e^{-.4F - .6128}$$

$\Rightarrow \ln(.5) = -.4F - .6128$

$\Rightarrow -.6931 = -.4F - .6128$
$\Rightarrow F = 0.20$

Question #156
Key: C

$$\left({}_{9}V + P\right)(1.03) = q_{x+9}b + \left(1 - q_{x+9}\right){}_{10}V$$
$$= q_{x+9}\left(b - {}_{10}V\right) + {}_{10}V$$
$$(343)(1.03) = 0.02904(872) + {}_{10}V$$
$$\Rightarrow {}_{10}V = 327.97$$
$$b = \left(b - {}_{10}V\right) + {}_{10}V = 872 + 327.97 = 1199.97$$
$$P = b\left(\frac{1}{\ddot{a}_x} - d\right) = 1200\left(\frac{1}{14.65976} - \frac{0.03}{1.03}\right)$$

= 46.92

${}_{9}V =$ initial reserve – P = 343 – 46.92 = 296.08

Question #157
Key: B

d = 0.06 $\Rightarrow$ *V* = 0.94

Step 1 Determine p_x

$$668 + 258vp_x = 1000vq_x + 1000v^2 p_x\left(p_{x+1} + q_{x+1}\right)$$
$$668 + 258(0.94)\,p_x = 1000(0.94)(1 - p_x) + 1000(0.8836)\,p_x(1)$$
$$668 + 242.52\,p_x = 940(1 - p_x) + 883.6\,p_x$$
$$p_x = 272/298.92 = 0.91$$

Step 2 Determine $1000P_{x:\overline{2}|}$

$$668 + 258(0.94)(0.91) = 1000P_{x:\overline{2}|}\left[1 + (0.94)(0.91)\right]$$
$$1000P_{x:\overline{2}|} = \frac{\left[220.69 + 668\right]}{1.8554} = 479$$

Question #158
Key: D

$$100{,}000(IA)^{1}_{40:\overline{10|}} = 100{,}000\, v\, p_{40}\left[(IA)^{1}_{41:\overline{10|}} - 10 v^{10}\, {}_{9}p_{41}\, q_{50}\right] + A^{1}_{40:\overline{10|}}(100{,}000) \quad [\text{see comment}]$$

$$= 100{,}000 \frac{0.99722}{1.06}\left[0.16736 - \frac{10\left(\frac{8{,}950{,}901}{9{,}287{,}264}\right)}{1.06^{10}} \times (0.00592)\right]$$

$$+ (0.02766 \times 100{,}000)$$

=15,513

Where $A^{1}_{40:\overline{10|}} = A_{40} - {}_{10}E_{40}\, A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$

$$= 0.02766$$

Comment: the first line comes from comparing the benefits of the two insurances. At each of age 40, 41, 42,…,49 $(IA)^{1}_{40:\overline{10|}}$ provides a death benefit 1 greater than $(IA)^{1}_{41:\overline{10|}}$. Hence the $A^{1}_{40:\overline{10|}}$ term. But $(IA)^{1}_{41:\overline{10|}}$ provides a death benefit at 50 of 10, while $(IA)^{1}_{40:\overline{10|}}$ provides 0. Hence a term involving ${}_{9|}q_{41} = {}_{9}p_{41}\, q_{50}$. The various *v*'s and *p*'s just get all actuarial present values at age 40.

Question #159
Key: A

$$1000\, {}_{1}V_{x} = \pi(1+i) - q_{x}(1000 - 1000\, {}_{1}V_{x})$$

$$40 = 80(1.1) - q_{x}(1000 - 40)$$

$$q_{x} = \frac{88 - 40}{960} = 0.05$$

$${}_{1}AS = \frac{(\text{G} - \text{ expenses})(1+i) - 1000 q_{x}}{p_{x}}$$

$$= \frac{(100 - (0.4)(100))(1.1) - (1000)(0.05)}{1 - 0.05}$$

$$= \frac{60(1.1) - 50}{0.95} = 16.8$$

Question #160
Key: C

At any age, $p_x'^{(1)} = e^{-0.02} = 0.9802$

$q_x'^{(1)} = 1 - 0.9802 = 0.0198$, which is also $q_x^{(1)}$, since decrement 2 occurs only at the end of the year.

Actuarial present value (APV) at the start of each year for that year's death benefits
= 10,000*0.0198 v = 188.1

$p_x^{(\tau)} = 0.9802 * 0.96 = 0.9410$

$E_x = p_x^{(\tau)} v = 0.941\ v = 0.941 * 0.95 = 0.8940$

APV of death benefit for 3 years $188.1 + E_{40} * 188.1 + E_{40} * E_{41} * 188.1 = 506.60$

Question #161
Key: B

$$\overset{\circ}{e}_{30:\overline{40}|} = \int_0^{40} {}_t p_{30} dt$$

$$= \int_0^{40} \frac{\omega - 30 - t}{\omega - 30} dt$$

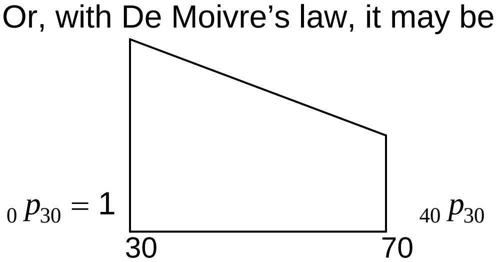
$$= t - \frac{t^2}{2(\omega - 30)} \Big|_0^{40}$$

$$= 40 - \frac{800}{\omega - 30}$$

$$= 27.692$$

$\omega = 95$

Or, with De Moivre's law, it may be simpler to draw a picture:



$$\overset{\circ}{e}_{30:\overline{40}|} = \text{area } = 27.692 = 40\frac{\left(1+{}_{40}p_{30}\right)}{2}$$

$$ {}_{40}p_{30} = 0.3846$$

$$\frac{\omega-70}{\omega-30} = 0.3846$$

$$\omega = 95$$

$${}_tp_{30} = \frac{65-t}{65}$$

$$Var = E\left(T\right)^2 - \left(E\left(T\right)\right)^2$$

One way to evaluate this expression is based on Equation 3.5.4 in Actuarial Mathematics

$$Var\left(T\right) = \int_0^\infty 2t\,{}_tp_x dt - \overset{\circ}{e}_x^2$$

$$= 2\int_0^{65} t\left(1-\frac{t}{65}\right)dt - \left(\int_0^{65}\left(1-\frac{t}{65}\right)dt\right)^2$$

$$= 2*\left(2112.5-1408.333\right)-\left(65-65/2\right)^2$$

$$= 1408.333-1056.25 = 352.08$$

Another way, easy to calculate for De Moivre's law is

$$Var\left(T\right) = \int_0^\infty t^2\,{}_tp_x\mu_x\left(t\right)dt - \left(\int_0^\infty t\,{}_tp_x\mu_x\left(t\right)dt\right)^2$$

$$= \int_0^{65} t^2 \times \frac{1}{65}dt - \left(\int_0^{65} t\times\frac{1}{65}dt\right)^2$$

$$= \frac{t^3}{3\times 65}\Bigg|_0^{65} - \left(\frac{t^2}{2\times 65}\Bigg|_0^{65}\right)^2$$

$$= 1408.33 - \left(32.5\right)^2 = 352.08$$

With De Moivre's law and a maximum future lifetime of 65 years, you probably didn't need to integrate to get $E\left(T\left(30\right)\right) = \overset{\circ}{e}_{30} = 32.5$

Likewise, if you realize (after getting $\omega = 95$) that $T\left(30\right)$ is uniformly on (0, 65), its variance is just the variance of a continuous uniform random variable:

$$Var = \frac{(65-0)^2}{12} = 352.08$$

Question #162
Key: E

$$_1V = \frac{218.15(1.06) - 10,000(0.02)}{1-0.02} = 31.88$$
$$_2V = \frac{(31.88 + 218.15)(1.06) - (9,000)(0.021)}{1-0.021} = 77.66$$

Question #163
Key: D

$$e_x = e_y = \sum_{k=1}^{\infty} {}_t p_x = 0.95 + 0.95^2 + \ldots$$
$$= \frac{0.95}{1-0.95} = 19$$
$$e_{xy} = p_{xy} + {}_2 p_{xy} + \ldots$$
$$= 1.02(0.95)(0.95) + 1.02(0.95)^2(0.95)^2 + \ldots$$
$$= 1.02\left[0.95^2 + 0.95^4 + \ldots\right] = \frac{1.02(0.95)^2}{1-0.95^2} = 9.44152$$
$$e_{\overline{xy}} = e_x + e_y - e_{xy} = 28.56$$

Question #164
Key: A

Local comes first. I board

So I get there first if he waits more than 28 – 16 = 12 minutes after the local arrived.

His wait time is exponential with mean 12
The wait before the local arrived is irrelevant; the exponential distribution is memoryless
Prob(exp with mean 12>12) $= e^{-12/12} = e^{-1} = 36.8\%$

Question #165
Key: E

Deer hit at time *s* are found by time *t* (here, *t* = 10) with probability *F*(*t* – *s*), where *F* is the exponential distribution with mean 7 days.
We can split the Poisson process "deer being hit" into "deer hit, not found by day 10" and "deer hit, found by day 10". By proposition 5.3, these processes are independent Poisson processes.
Deer hit, found by day 10, at time *s* has Poisson rate 20×*F*(*t* – *s*). The expected number hit and found by day 10 is its integral from 0 to 10.

$$E\left(N(t)\right) = 20\int_0^t F(t-s)\,ds$$

$$E\left(N(10)\right) = 20\int_0^{10} 1 - e^{\frac{-(10-s)}{7}}\,ds$$

$$= 20\left(10 - 7e^{\frac{s-10}{7}}\Big|_0^{10}\right)$$

$$= 20\left(10 - 7 + 7e^{-10/7}\right) = 94$$

Question #166
Key: E

$$\bar{a}_x = \int_0^\infty e^{-0.08t}dt = 12.5$$

$$\bar{A}_x = \int_0^\infty e^{-0.08t}(0.03)\,dt = \frac{3}{8} = 0.375$$

$$^2\bar{A}_x = \int_0^\infty e^{-0.13t}(0.03)\,dt = \frac{3}{13} = 0.23077$$

$$\sigma\left(\bar{a}_{\overline{T|}}\right) = \sqrt{\text{Var}\left[\bar{a}_{\overline{T|}}\right]} = \sqrt{\frac{1}{\delta^2}\left[{}^2\bar{A}_x - \left(\bar{A}_x\right)^2\right]} = \sqrt{400\left[0.23077 - (0.375)^2\right]} = 6.0048$$

$$\Pr\left[\bar{a}_{\overline{T|}} > \bar{a}_x - \sigma\left(\bar{a}_{\overline{T|}}\right)\right] = \Pr\left[\bar{a}_{\overline{T|}} > 12.5 - 6.0048\right]$$

$$= \Pr\left[\frac{1-v^T}{0.05} > 6.4952\right] = \Pr\left[0.67524 > e^{-0.05T}\right]$$

$$= \Pr\left[T > \frac{-\ln\, 0.67524}{0.05}\right] = \Pr\left[T > 7.85374\right]$$

$$= e^{-0.03\times 7.85374} = 0.79$$

Question #167
Key: A

$${}_5 p_{50}^{(\tau)} = e^{-(0.05)(5)} = e^{-0.25} = 0.7788$$

$${}_5 q_{55}^{(1)} = \int_0^5 \mu_{55}^{(1)}(t) \times e^{-(0.03+0.02)t} dt = -(0.02/0.05) e^{-0.05t} \Big|_0^5$$

$$= 0.4\left(1 - e^{-0.25}\right)$$

= 0.0885

Probability of retiring before $60 = {}_5 p_{50}^{(\tau)} \times {}_5 q_{55}^{(1)}$
= 0.7788*0.0885
= 0.0689

Question #168
Key: C

Complete the table:

$$l_{81} = l_{[80]} - d_{[80]} = 910$$

$l_{82} = l_{[81]} - d_{[81]} = 830$ (not really needed)

$$\mathring{e}_x = e_x + \frac{1}{2} \qquad \left(\frac{1}{2} \text{ since UDD}\right)$$

$$\mathring{e}_{[x]} = e_{[x]} + 1/2$$

$$\mathring{e}_{[x]} = \left[\frac{l_{81} + l_{82} + l_{83} + \dots}{l_{[80]}}\right] + \frac{1}{2}$$

$\left[\mathring{e}_{[80]} - \frac{1}{2}\right] l_{[80]} = l_{81} + l_{82} + \dots$ Call this equation (A)

$\left[\mathring{e}_{[81]} - \frac{1}{2}\right] l_{[81]} = l_{82} + \dots$ Formula like (A), one age later. Call this (B)

Subtract equation (B) from equation (A) to get

$$l_{81} = \left[\mathring{e}_{[80]} - \frac{1}{2}\right] l_{[80]} - \left[\mathring{e}_{[81]} - \frac{1}{2}\right] l_{[81]}$$

$$910 = [8.5 - 0.5]1000 - \left[\mathring{e}_{[81]} - 0.5\right]920$$

$$\mathring{e}_{[81]} = \frac{8000+460-910}{920} = 8.21$$

Alternatively, and more straightforward,

$$p_{[80]} = \frac{910}{1000} = 0.91$$
$$p_{[81]} = \frac{830}{920} = 0.902$$
$$p_{81} = \frac{830}{910} = 0.912$$

$$\mathring{e}_{[80]} = \frac{1}{2} q_{[80]} + p_{[80]}\left(1+\mathring{e}_{81}\right)$$

where $q_{[80]}$ contributes $\frac{1}{2}$ since UDD

$$8.5 = \frac{1}{2}(1-0.91)+(0.91)\left(1+\mathring{e}_{81}\right)$$

$$\mathring{e}_{81} = 8.291$$

$$\mathring{e}_{81} = \frac{1}{2} q_{81} + p_{81}\left(1+\mathring{e}_{82}\right)$$

$$8.291 = \frac{1}{2}(1-0.912)+0.912\left(1+\mathring{e}_{82}\right)$$

$$\mathring{e}_{82} = 8.043$$

$$\mathring{e}_{[81]} = \frac{1}{2} q_{[81]} + p_{[81]}\left(1+\mathring{e}_{82}\right)$$
$$= \frac{1}{2}(1-0.902)+(0.902)(1+8.043)$$
$$= 8.206$$

Or, do all the recursions in terms of *e*, not $\mathring{e}$, starting with $e_{[80]} = 8.5-0.5 = 8.0$, then final step $\mathring{e}_{[81]} = e_{[81]} + 0.5$

Question #169
Key: A

T	p_{x+t}	${}_t p_x$	v^t	$v^t\ {}_t p_x$
0	0.7	1	1	1
1	0.7	0.7	0.95238	0.6667
2	–	0.49	0.90703	0.4444
3	–	–	–	–

From above $\ddot{a}_{x:\overline{3}|} = \sum_{t=0}^{2} v^t\ {}_t p_x = 2.1111$

$$1000\ {}_2V_{x:\overline{3}|} = 1000\left(1 - \frac{\ddot{a}_{x+2:\overline{1}|}}{\ddot{a}_{x:\overline{3}|}}\right) = 1000\left(1 - \frac{1}{2.1111}\right) = 526$$

Alternatively,

$$P_{x:\overline{3}|} = \frac{1}{\ddot{a}_{x:\overline{3}|}} - d = 0.4261$$

$$\begin{aligned} 1000\ {}_2V_{x:\overline{3}|} &= 1000\left(v - P_{x:\overline{3}|}\right) \\ &= 1000\left(0.95238 - 0.4261\right) \\ &= 526 \end{aligned}$$

You could also calculate $A_{x:\overline{3}|}$ and use it to calculate $P_{x:\overline{3}|}$.

Question #170
Key: E

Let G be the expense-loaded premium.
Actuarial present value (APV) of benefits = $1000A_{50}$.

APV of expenses, except claim expense = $15+1\times\ddot{a}_{50}$

APV of claim expense = $50A_{50}$ (50 is paid when the claim is paid)

APV of premiums = $G\,\ddot{a}_{50}$

Equivalence principle: $G\ddot{a}_{50} = 1000A_{50} + 15 + 1\times\ddot{a}_{50} + 50A_{50}$

$$G = \frac{1050A_{50} + 15 + \ddot{a}_{50}}{\ddot{a}_{50}}$$

For De Moivre's with ω = 100, x = 50 $\quad A_{50} = \dfrac{a_{\overline{50}|}}{50} = 0.36512$

$$\ddot{a}_{50} = \frac{1-A_{50}}{d} = 13.33248$$

Solving for G, G = 30.88

Question #171
Key: A

$${}_{4}p_{50} = e^{-(0.05)(4)} = 0.8187$$

$${}_{10}p_{50} = e^{-(0.05)(10)} = 0.6065$$

$${}_{8}p_{60} = e^{-(0.04)(8)} = 0.7261$$

$${}_{18}p_{50} = \left({}_{10}p_{50}\right)\left({}_{8}p_{60}\right) = 0.6065\times 0.7261$$
$$= 0.4404$$

$${}_{4|14}q_{50} = {}_{4}p_{50} - {}_{18}p_{50} = 0.8187 - 0.4404 = 0.3783$$

Question #172
Key: D

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40}\,\ddot{a}_{45}$$
$$= 14.8166 - (0.73529)(14.1121)$$
$$= 4.4401$$

$$\pi\ddot{a}_{40:\overline{5}|} = 100,000 A_{45}\ v^5\ {}_5p_{40} + \pi\left(IA\right)^1_{40:\overline{5}|}$$
$$\pi = 100,000 A_{45} \times {}_5E_{40} / \left(\ddot{a}_{40:\overline{5}|} - \left(IA\right)^1_{40:\overline{5}|}\right)$$
$$= 100,000(0.20120)(0.73529)/(4.4401 - 0.04042)$$
$$= 3363$$

Question #173
Key: B

Calculate the probability that both are alive or both are dead.
P(both alive) = ${}_kp_{xy} = {}_kp_x \cdot {}_kp_y$
P(both dead) = ${}_kq_{\overline{xy}} = {}_kq_x\,{}_kq_y$
P(exactly one alive) = $1 - {}_kp_{xy} - {}_kq_{\overline{xy}}$
Only have to do two year's worth so have table

Pr(both alive)	Pr(both dead)	Pr(only one alive)
1	0	0
(0.91)(0.91) = 0.8281	(0.09)(0.09) = 0.0081	0.1638
(0.82)(0.82) = 0.6724	(0.18)(0.18) = 0.0324	0.2952

$$APV\ Annuity = 30,000\left(\frac{1}{1.05^0} + \frac{0.8281}{1.05^1} + \frac{0.6724}{1.05^2}\right) + 20,000\left(\frac{0}{1.05^0} + \frac{0.1638}{1.05^1} + \frac{0.2952}{1.05^2}\right) = 80,431$$

Alternatively,

$$\ddot{a}_{xy} = 1 + \frac{0.8281}{1.05} + \frac{0.6724}{1.05^2} = 2.3986$$
$$\ddot{a}_x = \ddot{a}_y = 1 + \frac{0.91}{1.05} + \frac{0.82}{1.05^2} = 2.6104$$
$$APV = 20,000\,\ddot{a}_x + 20,000\,\ddot{a}_y - 10,000\,\ddot{a}_{xy}$$

(it pays 20,000 if *x* alive and 20,000 if y alive, but 10,000 less than that if both are alive)

$$= (20,000)(2.6104) + (20,000)(2.6104) - (10,000)2.3986$$
$$= 80,430$$

Other alternatives also work.

Question #174
Key: C

Let *P* denote the contract premium.

$$P = \bar{a}_x = \int_0^{\infty} e^{-\delta t} e^{-\mu t} dt = \int_0^{\infty} e^{-0.05t} dt = 20$$
$$E[L] = \bar{a}_x^{IMP} - P$$
$$\bar{a}_x^{IMP} = \int_0^{10} e^{-0.03t} e^{-0.02t} dt + e^{-0.03(10)} e^{-0.02(10)} \int_0^{\infty} e^{-0.03t} e^{-0.01t} dt$$
$$= \frac{l - e^{-0.5}}{0.05} + \frac{e^{-0.5}}{0.04} = 23$$
$$E[L] = 23 - 20 = 3$$
$$\frac{E[L]}{P} = \frac{3}{20} = 15\%$$

Question #175
Key: C

$$A^{1}_{30:\overline{2}|} = 1000 v q_{30} + 500 v^2 \, {}_{1|}q_{30}$$
$$= 1000\left(\frac{1}{1.06}\right)(0.00153) + 500\left(\frac{1}{1.06}\right)^2 (0.99847)(0.00161)$$
$$= 2.15875$$

Initial fund $= 2.15875 \times 1000$ participants = 2158.75

Let F_n denote the size of Fund 1 at the end of year *n*.

$$F_1 = 2158.75(1.07) - 1000 = 1309.86$$
$$F_2 = 1309.86(1.065) - 500 = 895.00$$

Expected size of Fund 2 at end of year 2 = 0 (since the amount paid was the single benefit premium). Difference is 895.

Question #176
Key: C

$$Var[Z] = E\left[Z^2\right] - E[Z]^2$$

$$E[Z] = \int_0^\infty \left(v^t b_t\right) {}_t p_x \mu_x(t)\, dt = \int_0^\infty e^{-0.08t} e^{0.03t} e^{-0.02t}(0.02)\, dt$$

$$= \int_0^\infty (0.02) e^{-0.07t} dt = \frac{0.02}{0.07} = 2/7$$

$$E\left[Z^2\right] = \int_0^\infty \left(v_t\, b_t\right)^2 {}_t p_x \mu_x(t)\, dt = \int_0^\infty \left(e^{-0.05t}\right)^2 e^{-0.02t}(0.02)\, dt$$

$$= \int_0^\infty 0.02\, e^{-0.12t} \mu_x(t)\, dt = 2/12 = 1/6$$

$$Var[Z] = \frac{1}{6} - \left(2/7\right)^2 = \frac{1}{6} - \frac{4}{49} = 0.08503$$

Question #177
Key: C

From $A_x = 1 - d\,\ddot{a}x$ we have $A_x = 1 - \frac{0.1}{1.1}(8) = 3/11$

$$A_{x+10} = 1 - \frac{0.1}{1.1}(6) = 5/11$$

$$\bar{A}_x = A_x \times i/\delta$$

$$\bar{A}_x = \frac{3}{11} \times \frac{0.1}{\ln(1.1)} = 0.2861$$

$$\bar{A}_{x+10} = \frac{5}{11} \times \frac{0.1}{\ln(1.1)} = 0.4769$$

$${}_{10}V_x = \bar{A}_{x+10} - P\left(\bar{A}_x\right) \times \ddot{a}_{x+10}$$

$$= 0.4769 - \left(\frac{0.2861}{8}\right)6$$

= 0.2623

There are many other equivalent formulas that could be used.

Question #178
Key: C

Regular death benefit $= \int_0^\infty 100,000 \times e^{-0.06t} \times e^{-0.001t} 0.001 dt$

$$= 100,000 \left(\frac{0.001}{0.06 + 0.001} \right)$$

$$= 1639.34$$

Accidental death $= \int_0^{10} 100,000\, e^{-0.06t} e^{-0.001t} \left(0.0002\right) dt$

$$= 20 \int_0^{10} e^{-0.061t} dt$$

$$= 20 \left[\frac{1 - e^{-0.61}}{0.061} \right] = 149.72$$

Actuarial Present Value $= 1639.34 + 149.72 = 1789.06$

Question #179
Key: D

Once you are dead, you are dead. Thus, you never leave state 2 or 3, and rows 2 and 3 of the matrix must be (0 1 0) and (0 0 1).

Probability of dying from cause 1 within the year, given alive at age 61, is 160/800 = 0.20.

Probability of dying from cause 2 within the year, given alive at age 61, is 80/800 = 0.10

Probability of surviving to 62, given alive at 61, is 560/800 = 0.70 (alternatively, 1 – 0.20 – 0.10), so correct answer is D.

Question #180
Key: C

This first solution uses the method on the top of page 9 of the study note.

Note that if the species is it is not extinct after Q_3 it will never be extinct.
This solution parallels the example at the top of page 9 of the Daniel study note. We want the second entry of the product $(Q_1 \times Q_2 \times Q_3)e_3$ which is equal to $Q_1 \times (Q_2 \times (Q_3 \times e_3))$.

$$Q_3 \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0.1 \\ 1 \end{vmatrix}$$

$$Q_2 \begin{vmatrix} 0 \\ 0.1 \\ 1 \end{vmatrix} = \begin{vmatrix} 0.01 \\ 0.27 \\ 1 \end{vmatrix}$$

$$Q_1 \begin{vmatrix} 0.01 \\ 0.27 \\ 1 \end{vmatrix} = \begin{vmatrix} 0.049 \\ 0.489 \\ 1 \end{vmatrix}$$

The second entry is 0.489; that's our answer.

Alternatively, start with the row matrix (0 1 0) and project it forward 3 years.

$(0 \quad 1 \quad 0\ )\ Q_1 = (0.00 \quad 0.70 \quad 0.30)$
$(0 \quad 0.70 \quad 0.30)\ Q_2 = (0.07 \quad 0.49 \quad 0.44)$
$(0.07 \quad 0.49 \quad 0.44)\ Q_3 = (0.16 \quad 0.35 \quad 0.49)$

Thus, the probability that it is in state 3 after three transitions is 0.49.

Yet another approach would be to multiply $Q_1 \times Q_2 \times Q_3$, and take the entry in row 2, column 3. That would work but it requires more effort.

Question #181
Key: B

Probabilities of being in each state at time *t*:

t	Active	Disabled	Dead	Deaths
0	1.0	0.0	0.0	
1	0.8	0.1	0.1	0.1
2	0.65	0.15	0.2	0.1
3	not needed	not needed	0.295	0.095

We built the Active Disabled Dead columns of that table by multiplying each row times the transition matrix. E.g., to move from *t* = 1 to *t* = 2, (0.8 0.1 0.1) Q = (0.65 0.15 0.2)
The deaths column is just the increase in Dead. E.g., for t = 2, 0.2 – 0.1 = 0.1.

v = 0.9
APV of death benefits = $100,000*\left(0.1v+0.1v^2+0.095v^3\right)=24,025.5$

APV of $1 of premium = $1+0.8v+0.65v^2=2.2465$

$$\text{Benefit premium} = \frac{24,025.5}{2.2465}=10,695$$

Question #182
Key: A

Split into three independent processes:
Deposits, with $\lambda^*=(0.2)(100)(8)=160$ per day
Withdrawals, with $\lambda^*=(0.3)(100)(8)=240$ per day
Complaints. Ignore, no cash impact.

For aggregate deposits,

$$E(D)=(160)(8000)=1,280,000$$

$$Var(D)=(160)(1000)^2+(160)(8000)^2$$
$$=1.04\times10^{10}$$

For aggregate withdrawals

$$E(W)=(240)(5000)=1,200,000$$

$$Var(W)=(240)(2000)^2+(240)(5000)^2$$
$$=0.696\times10^{10}$$

$$E(W-D)=1,200,000-1,280,000=-80,000$$
$$Var(W-D)=0.696\times10^{10}+1.04\times10^{10}=1.736\times10^{10}$$
$$SD(W-D)=131,757$$
$$\begin{aligned}\Pr(W>D)=\Pr(W-D>0)=\Pr\left(\frac{W-D+80,000}{131,757}>\frac{80,000}{131,757}\right)\\ =1-\Phi(0.607)\\ =0.27\end{aligned}$$

Question #183
Key: D

Exponential inter-event times and independent implies Poisson process (imagine additional batteries being activated as necessary; we don't care what happens after two have failed).

Poisson rate of 1 per year implies failures in 3 years is Poisson with $\lambda=3$.

x	$f(x)$	$F(x)$
0	0.050	0.050
1	0.149	0.199

Probe works provided that there have been fewer than two failures, so we want $F(1)$ = 0.199.

Alternatively, the sum of two independent exponential $\theta=1$ random variables is Gamma with $\alpha=2,\quad\theta=1$

$$\begin{aligned}F(3)=\Gamma(2;3)&=\frac{1}{\Gamma(2)}\int_0^3 t\,e^{-t}dt\\ &=(-t-1)e^{-t}\Big|_0^3\\ &=1-4e^{-3}\\ &=0.80 \text{ is probability 2 have occurred}\end{aligned}$$

1 – 0.80 = 0.20

Question #184
Key: B

$$1000 P_{45} \ddot{a}_{45:\overline{15}|} + \pi\, \ddot{a}_{60:\overline{15}|} \times {}_{15}E_{45} = 1000 A_{45}$$

$$1000 \frac{A_{45}}{\ddot{a}_{45}} \left(\ddot{a}_{45} - {}_{15}E_{45}\, \ddot{a}_{60} \right) + \pi \left(\ddot{a}_{60} - {}_{15}E_{60}\, \ddot{a}_{75} \right)\left({}_{15}E_{45} \right) = 1000 A_{45}$$

$$\frac{201.20}{14.1121}\left(14.1121 - (0.72988)(0.51081)(11.1454)\right)$$
$$+\pi\left(11.1454 - (0.68756)(0.39994)(7.2170)\right) \times (0.72988)(0.51081) = 201.20$$

where ${}_{15}E_x$ was evaluated as ${}_5E_x \times {}_{10}E_{x+5}$

$$14.2573(9.9568) + (\pi)(3.4154) = 201.20$$

$$\pi = 17.346$$

Question #185
Key: A

$${}_1V = \left({}_0V + \pi \right)(1+i) - \left(1000 + {}_1V - {}_1V\right) q_x$$
$${}_2V = \left({}_1V + \pi \right)(1+i) - \left(2000 + {}_2V - {}_2V\right) q_{x+1} = 2000$$
$$\left(\left(\pi(1+i) - 1000 q_x\right) + \pi\right)(1+i) - 2000 q_{x+1} = 2000$$
$$\left(\left(\pi(1.08) - 1000 \times 0.1\right) + \pi\right)(1.08) - 2000 \times 0.1 = 2000$$

$$\pi = 1027.42$$

Question #186
Key: A

Let *Y* be the present value of payments to 1 person.
Let *S* be the present value of the aggregate payments.

$$E[Y] = 500\,\ddot{a}_x = 500\frac{(1-A_x)}{d} = 5572.68$$

$$\sigma_Y = \sqrt{\text{Var}[Y]} = \sqrt{(500)^2 \frac{1}{d^2}\left({}^2A_x - A_x^2\right)} = 1791.96$$

$$S = Y_1 + Y_2 + \ldots + Y_{250}$$

$$E(S) = 250E[Y] = 1{,}393{,}170$$

$$\sigma_S = \sqrt{250} \times \sigma_Y = 15.811388\sigma_Y = 28{,}333$$

$$0.90 = \Pr(S \le F) = \Pr\left[\frac{S - 1{,}393{,}170}{28{,}333} \le \frac{F - 1{,}393{,}170}{28{,}333}\right]$$

$$\approx \Pr\left[N(0,1) \le \frac{F - 1{,}393{,}170}{28{,}333}\right]$$

$$0.90 = \Pr\left(N(0,1) \le 1.28\right)$$

$$F = 1{,}393{,}170 + 1.28(28{,}333)$$

=1.43 million

Question #187
Key: A

$$q_{41}'^{(1)} = 1 - p_{41}'^{(1)} = 1 - \left(p_{41}{}^{(\tau)}\right)^{q_{41}^{(1)} / q_{41}^{(\tau)}}$$

$$l_{41}{}^{(\tau)} = l_{40}{}^{(\tau)} - d_{40}{}^{(1)} - d_{40}{}^{(2)} = 1000 - 60 - 55 = 885$$

$$d_{41}{}^{(1)} = l_{41}{}^{(\tau)} - d_{41}{}^{(2)} - l_{42}{}^{(\tau)} = 885 - 70 - 750 = 65$$

$$p_{41}{}^{(\tau)} = \frac{750}{885} \qquad \frac{q_{41}{}^{(1)}}{q_{41}{}^{(\tau)}} = \frac{65}{135}$$

$$q_{41}'^{(1)} = 1 - \left(\frac{750}{885}\right)^{65/135} = 0.0766$$

Question #188
Key: D

$$s(x) = \left(1 - \frac{x}{\omega}\right)^{\alpha}$$

$$\mu(x) = \frac{d}{dx}\ \log\ \left(s(x)\right) = \frac{\alpha}{\omega - x}$$

$$\overset{\circ}{e}_x = \int_0^{\omega - x} \left(1 - \frac{t}{\omega - x}\right)^{\alpha} dt = \frac{\omega - x}{\alpha + 1}$$

$$\overset{\circ}{e}_0^{\text{new}} = \frac{1}{2} \times \frac{\omega}{\alpha^{\text{old}} + 1} = \frac{\omega}{\alpha^{\text{new}} + 1} \Rightarrow \alpha^{\text{new}} = 2\alpha^{\text{old}} + 1$$

$$\mu_0^{(\text{new})} = \frac{2\alpha^{\text{old}} + 1}{\omega} = \frac{9}{4} \times \frac{\alpha^{\text{old}}}{\omega} \Rightarrow \alpha^{\text{old}} = 4$$

Question #189
Key: C

Constant force implies exponential lifetime

$$Var[T] = E\left[T^2\right] - \left(E[T]\right)^2 = \frac{2}{\mu^2} - \left(\frac{1}{\mu}\right)^2 = \frac{1}{\mu^2} = 100$$

$$\mu = 0.1$$

$$\begin{aligned} E\left[\min(T,10)\right] &= \int_0^{10} t(0.1)e^{-.1t}dt + \int_{10}^{\infty} 10(0.1)e^{-.1t}dt \\ &= -te^{-.1t} - 10e^{-.1t}\Big|_0^{10} - 10e^{-.1t}\Big|_{10}^{\infty} \\ &= -10e^{-1} - 10e^{-1} + 10 + 10e^{-1} \\ &= 10\left(1 - e^{-1}\right) = 6.3 \end{aligned}$$

Question #190

Key: A

$$G\ddot{a}_{x:\overline{15}|} = 100,000A_x + \overbrace{\left(0.08G + 0.02G\ddot{a}_{x:\overline{15}|}\right)}^{\text{\% premium amount for 15 years}} + \underbrace{\left((x-5) + 5\ddot{a}_x\right)}_{\text{Per policy for life}}$$

$$4669.95(11.35) = 51,481.97 + (0.08)(4669.95) + (0.02)(11.35)(4669.95) + \left((x-5) + 5\ddot{a}_x\right)$$

$$\ddot{a}_x = \frac{1 - Ax}{d} = \frac{1 - 0.5148197}{0.02913} = 16.66$$

$$53,003.93 = 51,481.97 + 1433.67 + (x-5) + 83.30$$
$$4.99 = (x-5)$$
$$x = 9.99$$

The % of premium expenses could equally well have been expressed as $0.10G + 0.02G\,a_{x:\overline{14}|}$.
The per policy expenses could also be expressed in terms of an annuity-immediate.

Question #191
Key: D

For the density where $T(x) \neq T(y)$,

$$\Pr\left(T(x) < T(y)\right) = \int_{y=0}^{40}\int_{x=0}^{y} 0.0005dxdy + \int_{y=40}^{50}\int_{x=0}^{40} 0.0005dxdy$$

$$= \int_{y=0}^{40} 0.0005x\Big|_0^y dy + \int_{y=40}^{50} 0.0005x\Big|_0^{40} dy$$

$$= \int_0^{40} 0.0005ydy + \int_{y=40}^{50} 0.02\,dy$$

$$= \frac{0.0005y^2}{2}\Big|_0^{40} + 0.02y\Big|_{40}^{50}$$

$$= 0.40 + 0.20 = 0.60$$

For the overall density,

$\Pr\left(T(x) < T(y)\right) = 0.4 \times 0 + 0.6 \times 0.6 = 0.36$

where the first 0.4 is the probability that $T(x) = T(y)$ and the first 0.6 is the probability that $T(x) \neq T(y)$.

Question #192
Key: B

The conditional expected value of the annuity, given μ, is $\dfrac{1}{0.01+\mu}$.

The unconditional expected value is

$$\bar{a}_x = 100\int_{0.01}^{0.02} \frac{1}{0.01+\mu} d\mu = 100 \ln\left(\frac{0.01+0.02}{0.01+0.01}\right) = 40.5$$

100 is the constant density of μ on the internal $[0.01, 0.02]$. If the density were not constant, it would have to go inside the integral.

Question #193
Key: E

Recall $\quad \overset{\circ}{e}_x = \dfrac{\omega - x}{2}$

$$\overset{\circ}{e}_{\overline{x:x}} = \overset{\circ}{e}_x + \overset{\circ}{e}_x - \overset{\circ}{e}_{x:x}$$

$$\overset{\circ}{e}_{x:x} = \int_0^{\omega-x}\left(1-\frac{t}{\omega-x}\right)\left(1-\frac{t}{\omega-y}\right)dt$$

Performing the integration we obtain

$$\overset{\circ}{e}_{x:x} = \frac{\omega - x}{3}$$

$$\overset{\circ}{e}_{\overline{x:x}} = \frac{2(\omega - x)}{3}$$

(i) $\quad \dfrac{2(\omega-2a)}{3} = 3 \times \dfrac{2(\omega-3a)}{3} \Rightarrow 2\omega = 7a$

(ii) $\quad \dfrac{2}{3}(\omega - a) = k \times \dfrac{2(\omega-3a)}{3}$

$\quad 3.5a - a = k(3.5a - 3a)$

$k = 5$

The solution assumes that all lifetimes are independent.

Question #194
Key: B

Upon the first death, the survivor receives $10,000\frac{\mu}{\mu+\delta} = 10,000\left(\frac{0.10}{0.10+0.04}\right) = 7143$

The actuarial present value of the insurance of 7143 is

$$7,143\frac{\mu_{xy}}{\mu_{xy}+\delta} = (7,143)\left(\frac{0.12}{0.12+0.04}\right) = 5357$$

If the force of mortality were not constant during each insurance period, integrals would be required to express the actuarial present value.

Question #195
Key: E

Let ${}_k p_0 =$ Probability someone answers the first *k* problems correctly.

$${}_2 p_0 = (0.8)^2 = 0.64 \qquad {}_4 p_0 = (0.8)^4 = 0.41$$

$${}_2 p_{0:0} = ({}_2 p_0)^2 = 0.64^2 = 0.41 \qquad {}_4 p_{0:0} = (0.41)^2 = 0.168$$

$${}_2 p_{\overline{0:0}} = {}_2 p_0 + {}_2 p_0 - {}_2 p_{0:0} = 0.87 \qquad {}_4 p_{\overline{0:0}} = 0.41 + 0.41 - 0.168 = 0.652$$

Prob(second child loses in round 3 or 4) = ${}_2 p_{\overline{0:0}} - {}_4 p_{\overline{0:0}}$
= 0.87-0.652
= 0.218

Prob(second loses in round 3 or $4|$ second loses after round 2) = $\frac{{}_2 p_{\overline{0:0}} - {}_4 p_{\overline{0:0}}}{{}_2 p_{\overline{0:0}}}$

$$= \frac{0.218}{0.87} = 0.25$$

Question #196
Key: E

If (40) dies before 70, he receives one payment of 10, and *Y* = 10. Under DeMoivre, the probability of this is (70 – 40)/(110 – 40) = 3/7

If (40) reaches 70 but dies before 100, he receives 2 payments.

$Y = 10 + 20v^{30} = 16.16637$

The probability of this is also 3/7. (Under DeMoivre, all intervals of the same length, here 30 years, have the same probability).

If (40) survives to 100, he receives 3 payments.

$Y = 10 + 20v^{30} + 30v^{60} = 19.01819$

The probability of this is 1 – 3/7 – 3/7 = 1/7

$E(Y) = (3/7)\times 10 + (3/7)\times 16.16637 + (1/7)\times 19.01819 = 13.93104$

$E(Y^2) = (3/7)\times 10^2 + (3/7)\times 16.16637^2 + (1/7)\times 19.01819^2 = 206.53515$

$Var(Y) = E(Y^2) - [E(Y)]^2 = 12.46$

Since everyone receives the first payment of 10, you could have ignored it in the calculation.

Question #197
Key: C

$$\begin{aligned} \mathrm{E}(Z) &= \sum_{k=0}^{2} \left(v^{k+1} b_{k+1}\right) \; {}_k p_x \; q_{x+k} \\ &= \left[v(300)\times 0.02 + v^2 (350)(0.98)(0.04) + v^3 400(0.98)(0.96)(0.06) \right] \\ &= 36.8 \end{aligned}$$

$$\begin{aligned} E(Z^2) &= \sum_{k=0}^{2} \left(v^{k+1} b_{k+1}\right)^2 \; {}_k p_x \; q_{x+k} \\ &= v^2 (300)^2 \times 0.02 + v^4 (350)^2 (0.98)(0.04) + v^6 400^2 (0.98)(0.96) 0.06 \\ &= 11,773 \end{aligned}$$

$$\begin{aligned} Var[Z] &= E(Z^2) - E(Z)^2 \\ &= 11,773 - 36.8^2 \\ &= 10,419 \end{aligned}$$

Question #198
Key: A

	Benefits +	Expenses	– Premiums
${}_0L_e =$	$1000v^3$ +	$(0.20G+8)+(0.06G+2)v+(0.06G+2)v^2$	$-\ G\ddot{a}_{\overline{3}|}$

at $G = 41.20$ and $i = 0.05$,

${}_0L_e\ (\text{for } K = 2) = 770.59$

Question #199
Key: D

$P = 1000P_{40}$

$(235+P)(1+i)-0.015(1000-255)=255$ [A]

$(255+P)(1+i)-0.020(1000-272)=272$ [B]

Subtract [A] from [B]

$$20(1+i)-3.385=17$$

$$1+i=\frac{20.385}{20}=1.01925$$

Plug into [A] $(235+P)(1.01925)-0.015(1000-255)=255$

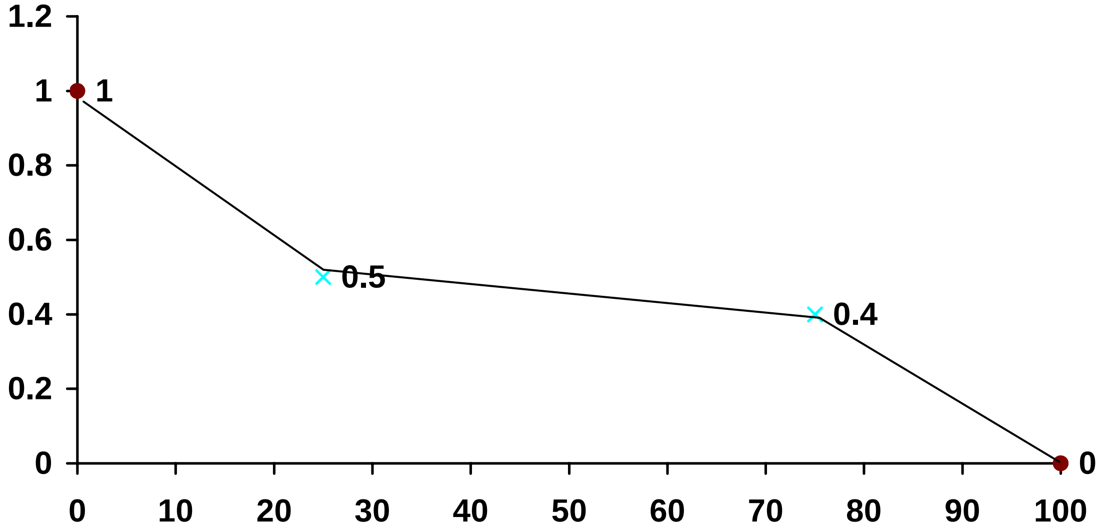
$$235+P=\frac{255+11.175}{1.01925}$$

$$P=261.15-235=26.15$$

$$1000\ {}_{25}V_{40}=\frac{(272+26.15)(1.01925)-(0.025)(1000)}{1-0.025}$$

$$=286$$

Question #200
Key: A



	Given		Given		Given		Given
x	0	15	25	35	75	90	100
$s(x)$	1	0.70	0.50	0.48	0.4	0.16	0
		Linear Interpolation		Linear Interpolation		Linear Interpolation	

$$_{55}q_{35} = 1 - \frac{s(90)}{s(35)} = 1 - \frac{0.16}{0.48} = \frac{32}{48} = 0.6667$$

$$_{20|55}q_{15} = \frac{s(35) - s(90)}{s(15)} = \frac{0.48 - 0.16}{0.70} = \frac{32}{70} = 0.4571$$

$$\frac{_{20|55}q_{15}}{_{55}q_{35}} = \frac{0.4571}{0.6667} = 0.6856$$

Alternatively,

$$\frac{_{20|55}q_{15}}{_{55}q_{35}} = \frac{_{20}p_{15} \times {}_{55}q_{35}}{_{55}q_{35}} = {}_{20}p_{15} = \frac{s(35)}{s(15)}$$

$$= \frac{0.48}{0.70}$$

$$= 0.6856$$

Question #201
Key: A

$$s(80) = \tfrac{1}{2} * \left(e \wedge (-0.16 * 50) + e \wedge (-0.08 * 50)\right) = 0.00932555$$
$$s(81) = \tfrac{1}{2} * \left(e \wedge (-0.16 * 51) + e \wedge (-0.08 * 51)\right) = 0.008596664$$

$$p_{80} = s(81)/s(80) = 0.008596664/0.00932555 = 0.9218$$
$$q_{80} = 1 - 0.9218 = 0.078$$

Alternatively (and equivalent to the above)

For non-smokers, $p_x = e^{-0.08} = 0.923116$

$_{50}p_x = 0.018316$

For smokers, $p_x = e^{-0.16} = 0.852144$

$_{50}p_x = 0.000335$

So the probability of dying at 80, weighted by the probability of surviving to 80, is

$$\frac{0.018316 \times (1 - 0.923116) + 0.000335 \times (1 - 0.852144)}{0.018316 + 0.000335} = 0.078$$

Question #202
Key: B

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	2000	20	60
41	1920	30	50
42	1840	40	

because $2000 - 20 - 60 = 1920$; $1920 - 30 - 50 = 1840$

Let premium = *P*

APV premiums = $\left(\frac{2000}{2000} + \frac{1920}{2000}v + \frac{1840}{2000}v^2\right)P = 2.749P$

APV benefits = $1000\left(\frac{20}{2000}v + \frac{30}{2000}v^2 + \frac{40}{2000}v^3\right) = 40.41$

$$P = \frac{40.41}{2.749} = 14.7$$

Question #203
Key: A

$$\bar{a}_{30} = \int_0^{10} e^{-0.08t}\, e^{-0.05} dt + {}_{10}E_x \int_0^{\infty} e^{-0.08t} e^{-0.08t} dt$$
$$= \int_0^{10} e^{-0.13t} dt + e^{-1.3} \int_0^{\infty} e^{-0.16} dt$$
$$\frac{-e^{-0.13t}}{0.13}\Big|_0^{10} + \left(e^{-1.3}\right) \frac{-e^{-0.16t}}{0.16}\Big|_0^{\infty}$$
$$= \frac{-e^{1.3}}{0.13} + \frac{1}{0.13} + \frac{e^{-1.3}}{0.16}$$
$$= 7.2992$$
$$\bar{A}_{30} = \int_0^{10} e^{-0.08t} e^{-0.05t} \left(0.05\right) dt + e^{-1.3} \int_0^{\infty} e^{-0.16t} \left(0.08\right) dt$$
$$= 0.05\left(\frac{1}{0.13} - \frac{e^{-1.3}}{0.13}\right) + \left(0.08\right) \frac{e^{-1.3}}{0.16}$$
$$= 0.41606$$
$$= \bar{P}\left(\bar{A}_{30}\right) = \frac{\bar{A}_{30}}{\bar{a}_{30}} = \frac{0.41606}{7.29923} = 0.057$$
$$\bar{a}_{40} = \frac{1}{0.08 + 0.08} = \frac{1}{0.16}$$
$$\bar{A}_{40} = 1 - \delta\, \bar{a}_{40}$$
$$= 1 - \left(0.08 / 0.16\right) = 0.5$$
$${}_{10}\bar{V}\left(\bar{A}_{40}\right) = \bar{A}_{40} - \bar{P}\left(\bar{A}_{40}\right) \bar{a}_{40}$$
$$= 0.5 - \frac{\left(0.057\right)}{0.16} = 0.14375$$

Question #204
Key: C

Let T be the future lifetime of Pat, and [T] denote the greatest integer in T. ([T] is the same as K, the curtate future lifetime).

$$L = 100,000\, v^T - 1600\, \ddot{a}_{\overline{[T]+1}|} \qquad 0 < T \le 10$$
$$= 100,000 v^T - 1600\, \ddot{a}_{\overline{10}|} \qquad 10 < t \le 20$$
$$-1600\, \ddot{a}_{\overline{10}|} \qquad 20<t$$

Minimum is $-1600\, \ddot{a}_{\overline{10}|}$ when evaluated at i = 0.05

$$= -12,973$$

Question #205
Key: B

Method 1: as three independent processes, based on the amount deposited. Within each
process, since the amount deposited is always the same, $Var(X)=0$.

Rate of depositing 10 = 0.05 * 22 = 1.1
Rate of depositing 5 = 0.15 * 22 = 3.3
Rate of depositing 1 = 0.80 * 22 = 17.6

Variance of depositing 10 = 1.1 * 10 * 10 = 110
Variance of depositing 5 = 3.3 * 5 * 5 = 82.5
Variance of depositing 1 = 17.6 * 1 *1 = 17.6

Total Variance = 110 + 82.5 + 17.6 = 210.1

Method 2: as a single compound Poisson process

$$E(X)=0.8\times1+0.15\times5+0.05\times10=2.05$$

$$E(X^2)=0.8\times1^2+0.15\times5^2+0.05\times10^2=9.55$$

$$\begin{aligned}Var(S)&=E(N)Var(X)+Var(N)(E(X))^2\\&=(22)(5.3475)+(22)(2.05^2)\\&=210.1\end{aligned}$$

Question #206
Key: A

$P\ddot{a}_{x:\overline{3}|}=APV$ (stunt deaths)

$$P\left[\frac{2500+2486/1.08+2466/(1.08)^2}{2500}\right]=500000\left(\frac{4/1.08+5/(1.08)^2+6/(1.08)^3}{2500}\right)$$

$$P(2.77)=2550.68$$

$$\Rightarrow p=921$$

Question #207
Key: D

$$\mathring{e}_{30:\overline{50|}} = \frac{\int_{30}^{80} s(x)\,dx}{s(30)} = \frac{\int_{30}^{80}\left(1-\frac{x^2}{10,000}\right)dx}{1-\left(\frac{30}{100}\right)^2}$$

$$= \frac{\left(x-\frac{x^3}{30,000}\right)\Big|_{30}^{80}}{0.91}$$

$$= \frac{33.833}{0.91}$$

$$= 37.18$$

Question #208
Key: B

$$A_{60} = v \times \left(p_{60} \times A_{61} + q_{60}\right)$$

$$= (1/1.06) \times (0.98 \times 0.440 + 0.02)$$

$$= 0.42566$$

$$\ddot{a}_{60} = (1 - A_{60})/d$$

$$= (1-0.42566)/(0.06/1.06)$$

$$= 10.147$$

$$1000\,{}_{10}V_{50} = 1000A_{60} - 1000P_{50} \times \ddot{a}_{60}$$

$$= 425.66 - 10.147 \times 25$$

$$= 172$$

Question #209
Key: E

Let Portfolio be the present value random variable for the aggregate payments.

Let $Y_{65} =$ present value random variable for an annuity due of one on one life age 65.
Thus $E\left(Y_{65}\right) = \ddot{a}_{65}$
Let $Y_{75} =$ present value random variable for an annuity due of one on one life age 75.
Thus $E\left(Y_{75}\right) = \ddot{a}_{75}$

Let *X* represent the 95th percentile.

E (Portfolio) $= 50(2)\ddot{a}_{65} + 30(1)\ddot{a}_{75}$
$$= 100(9.8969) + 30(7.217) = 1206.20$$
Var (Portfolio) $= 50 \times 2^2 Var\left[Y_{65}\right] + 30(1)^2 Var\left[Y_{75}\right] = 200(13.2996) + 30(11.5339) = 3005.94$

where $Var\left[Y_{65}\right] = \dfrac{1}{d^2}\left({}^2A_{65} - A_{65}^2\right) = \dfrac{1}{(0.05660)^2}\left[0.23603 - (0.4398)^2\right] = 13.2996$

and $Var\left[Y_{75}\right] = \dfrac{1}{d^2}\left({}^2A_{75} - A_{75}^2\right) = \dfrac{1}{(0.05660)^2}\left[0.38681 - (0.59149)^2\right] = 11.5339$

$$\Pr\left[\left(\frac{X - E(\text{Portfolio})}{\sqrt{Var(\text{Portfolio})}}\right) \le 1.645\right] = 0.95 \Rightarrow X = E(\text{Portfolio}) + 1.645\sqrt{Var[\text{Portfolio}]}$$
$$= 1206.20 + 1.645(54.826)$$
$$= 1296.39$$

Question #210
Key: C

$$\overline{a} = \int_0^\infty e^{-\delta t} \times e^{-\mu t} dt = \frac{1}{\delta + \mu}$$
$$APV = 50{,}000 \times \frac{1}{0.5}\int_{0.5}^{1} \frac{1}{\delta + \mu} d\mu = 100{,}000 \times \left[\ln(\delta + 1) - \ln(\delta + 0.5)\right]$$
$$= 100{,}000 \times \ln\left(\frac{0.045 + 1}{0.045 + 0.5}\right)$$
$$= 65{,}099$$

Question #211
Key: E

The process described, where a key feature is the exponential time between events, is a
Poisson process with $\lambda = 1/5$ per minute.
The number of claims in any interval of length *n* minutes has a Poisson distribution with mean
$\lambda n = n/5$.

Here $n = 10$. So parameter = 10/5 = 2

$$\Pr(N \geq 2) = 1 - \Pr(N = 0) - \Pr(N = 1)$$
$$= 1 - e^{-2} - e^{-2}2$$
$$= 1 - 0.135 - 0.271 = 0.594$$

Question #212
Key: D

The payouts in any time period of length *t* have a Poisson distribution with parameter $5t$.

The payouts can be grouped by size. For each *i*, the number of payouts of size *i* is a Poisson random variable with mean $5t/2^i$, and these random variables are independent.
Since they are independent Poisson random variables, the sum of the payouts of size 1, 2 or 3 is a Poisson random variable with mean $\left(\frac{5t}{2} + \frac{5t}{4} + \frac{5t}{8}\right) = \frac{35t}{8}$

For $t = 1/3$ hour, the mean is $\frac{35}{8} \times \frac{1}{3} = 1.4583$

$$f(0) = e^{-1.4583} = 0.23$$

Question #213
Key: D

How long was the expected wait during first 45 minutes? In that interval, wait is exponential with
$\theta = 30$, so

$$E\left[\min\left(X,45\right)\right] = \int_0^{45} x \frac{1}{30} e^{-\frac{x}{30}} dx + \int_{45}^{\infty} 45 \frac{1}{30} e^{-\frac{x}{30}} dx$$
$$= 30\left(1 - e^{-\frac{45}{30}}\right) = 23.31$$

Expected trains $= \frac{45}{30} = 1.5$, so $f\left(0 \text{ trains}\right) = \frac{e^{-1.5}\left(1.5\right)^0}{0!} = 0.223$

If 0, wait an additional 15 minutes (expected) so

Total expected wait $= 23.31 + \left(0.223\right)\left(15\right) = 26.65$

Note that this problem is equivalent to calculate $\overset{\circ}{e}_x$

where $\mu_x\left(t\right) = \begin{cases} 1/30, & 0 \le t < 45 \\ 1/15, & t \ge 45 \end{cases}$

and solution is $\overset{\circ}{e}_x = \overset{\circ}{e}_{x:\overline{45}|} + {}_{45}p_x \, \overset{\circ}{e}_{x+45}$

Question #214
Key: A

Let π be the benefit premium at issue.

$$\pi = 10{,}000 \frac{A_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}} = 10{,}000 \frac{\left[0.20120 - 0.25634\left(0.43980\right) + 0.25634\right]}{14.1121 - 0.25634\left(9.8969\right)}$$
$$= 297.88$$

The expected prospective loss at age 60 is

$$10{,}000\,{}_{15}V_{45:\overline{20}|} = 10{,}000 A_{60:\overline{5}|} - 297.88\,\ddot{a}_{60:\overline{5}|}$$
$$= 10{,}000 \times 0.7543 - 297.88 \times 4.3407$$
$$= 6250$$

where $A^{1}_{60:\overline{5}|} = 0.36913 - 0.68756(0.4398) = 0.06674$

$A_{60:\overset{1}{\overline{5}|}} = 0.68756$

$A_{60:\overline{5}|} = 0.06674 + 0.68756 = 0.7543$

$\ddot{a}_{60:\overline{5}|} = 11.1454 - 0.68756 \times 9.8969 = 4.3407$

After the change, expected prospective loss = $10{,}000 A^{1}_{60:\overline{5}|} + (\text{Reduced Amount})\, A_{60:\overset{1}{\overline{5}|}}$

Since the expected prospective loss is the same

$6250 = (10{,}000)(0.06674) + (\text{Reduced Amount})(0.68756)$

Reduced Amount = 8119

Question #215
Key: D

$$\bar{A}_x = \bar{A}^{1}_{x:\overline{5}|} + {}_5E_x\, \bar{A}^{\;1}_{x+5:\overline{7}|} + {}_{12}E_x\, \bar{A}_{x+12}$$

where

$${}_5E_x = e^{-5(0.04+0.02)} = 0.7408$$
$$\bar{A}^{1}_{x:\overline{5}|} = \frac{0.04}{0.04+0.02} \times (1 - 0.7408) = 0.1728$$
$${}_7E_{x+5} = e^{-7(0.05+0.02)} = 0.6126$$
$$\bar{A}^{\;1}_{x+5:\overline{7}|} = \left(\frac{0.05}{0.05+0.02}\right)(1 - 0.6126) = 0.2767$$
$${}_{12}E_x = {}_5E_x \times {}_7E_{x+5} = 0.7408 \times 0.6126 = 0.4538$$
$$\bar{A}_{x+12} = \frac{0.05}{0.05+0.03} = 0.625$$
$$\bar{A}_x = 0.1728 + (0.7408)(0.2767) + (0.4538)(0.625)$$
= 0.6614

Question #216
Key: A

APV of Accidental death benefit and related settlement expense =

$$(2000 \times 1.05) \times \frac{0.004}{0.004 + 0.04 + 0.05} = 89.36$$

APV of other DB and related settlement expense = $(1000 \times 1.05) \times \dfrac{0.04}{0.094} = 446.81$

APV of Initial expense = 50

APV of Maintenance expense = $\dfrac{3}{0.094} = 31.91$

APV of future premiums $= \dfrac{100}{0.094} = 1063.83$

APV of ${}_0L_e = 89.36 + 446.81 + 50 + 31.91 - 1063.83$

$= -445.75$

Question #217
Key: C

Compute the probabilities of moving from healthy to NH. There are three paths.

H to H to NH: (0.8)(0.05) = 0.04
H to HHC to NH: (0.15)(0.05) = 0.0075
H to NH to NH: (0.05)(1) = 0.05

Summing, we get 0.0975 as the probability for each member.
Variance for *m* members = *mpq*, here = 50*(0.0975)(0.9025) = 4.40

Question #218
Key: C

$$Q_0 = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad Q_0 \times Q_1 = \begin{pmatrix} 0.36 & 0.18 & 0.46 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_0 \times Q_1 \times Q_2 = \begin{pmatrix} 0 & 0.108 & 0.892 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{APV}(\text{Premiums}) = 1 + 0.9v + 0.54v^2 + 0.108v^3 = 2.35$$

$$\text{APV}(\text{Benefits}) = 4\left(0.3v + 0.18v^2 + 0.108v^3\right) = 2.01$$

Difference = 2.35 – 2.01 = 0.34

In the formula for APV (Premiums), states 0 and 1 are combined. For example, the $0.54v^2$ term represents a 0.36 probability of being in state 0 plus a 0.18 probability of being in
state 1.

Alternatively, the same effort here but often shorter when everyone is in the same initial state:

$$\begin{pmatrix}1.00 & 0.00 & 0.00\end{pmatrix}\times Q_0 = \begin{pmatrix}0.6 & 0.3 & 1\end{pmatrix}$$
$$\begin{pmatrix}0.60 & 0.30 & 0.10\end{pmatrix}\times Q_1 = \begin{pmatrix}0.36 & 0.18 & 0.46\end{pmatrix}$$
$$\begin{pmatrix}0.36 & 0.18 & 0.46\end{pmatrix}\times Q_2 = \begin{pmatrix}0 & 0.108 & 0.892\end{pmatrix}$$

This method just calculates the top row of the cumulative transition matrix. It gives the same elements you use if you calculate the complete cumulative transition matrix, so you finish the problem the same way as before.

Question #219
Key: E

$${}_{0.25|1.5}q_x = {}_{0.25}p_x - {}_{1.75}p_x$$

Let μ be the force of mortality in year 1, so 3μ is the force of mortality in year 2.
Probability of surviving 2 years is 10%

$$\begin{cases} 0.10 = p_x \; p_{x+1} = e^{-\mu}e^{-3\mu} = e^{-4\mu} \\ \mu = \dfrac{\ln(0.1)}{4} = 0.5756 \end{cases}$$

$${}_{0.25}p_x = e^{-\frac{1}{4}(0.5756)} = 0.8660$$

$${}_{1.75}p_x = p_x \times {}_{0.75}p_{x+1} = e^{-\mu}e^{-\frac{3}{4}(3\mu)} = e^{-\frac{13}{4}(0.5756)} = 0.1540$$

$${}_{1.5}q_{x+0.25} = \frac{{}_{0.25|1.5}q_x}{{}_{0.25}p_x} = \frac{{}_{0.25}p_x - {}_{1.75}p_x}{{}_{0.25}p_x} = \frac{0.866 - 0.154}{0.866} = 0.82$$

Question #220
Key: C

The form of l_x for non-smokers matches DeMoivre's law, so

$$\mu_x^{NS} = \frac{1}{110-x}$$
$$= \tfrac{1}{2}\,\mu_x^S \Rightarrow \mu_x^S = \frac{2}{110-x}$$
$$\Rightarrow l_x^S = l_0^S\,(110-x)^2$$ [see note below]

Thus $\quad {}_t p_{20}^S = \frac{l_{20+t}^S}{l_{20}^S} = \frac{(90-t)^2}{90^2}$

$${}_t p_{25}^{NS} = \frac{l_{25+t}^{NS}}{l_{25}^{NS}} = \frac{(85-t)}{85}$$

$$\begin{aligned}
\mathring{e}_{20:25} &= \int_0^{85} {}_t p_{20:25} dt \\
&= \int_0^{85} {}_t p_{20}^S \; {}_t p_{25}^{NS} dt = \int_0^{85} \frac{(90-t)^2}{(90)^2} \frac{(85-t)}{85} dt \\
&= \frac{1}{688,500} \int_0^{85} (90-t)^2 (90-t-5) dt \\
&= \frac{1}{688,500} \left[\int_0^{85} (90-t)^3 dt - 5\int_0^{85} (90-t)^2 dt \right. \\
&= \frac{1}{688,500} \left[\frac{-(90-t)^4}{4} + \frac{5(90-t)^3}{3} \right]_0^{85} \\
&= \frac{1}{688,500} \left[-156.25 + 208.33 + 16,402,500 - 1,215,000 \right]
\end{aligned}$$

= 22.1

[There are other ways to evaluate the integral, leading to the same result].

Note: the solution above assumes the candidate will recognize that the smoker mortality is modified DeMoivre and can proceed directly to the l_x or $s(x)$ form. The $s(x)$ form is derived as $s(x) = e^{-\int_0^x \left(\frac{2}{110-t}\right) dt} = e^{2\ln(110-t)\Big|_0^x} = \left(\frac{110-x}{110}\right)^2$

The l_x form is equivalent.

Question #221
Key: B

$\ddot{a}_{30:\overline{20|}} = \ddot{a}_{30:\overline{10|}} + {}_{10}E_{30} \times \ddot{a}_{40:\overline{10|}}$

$15.0364 = 8.7201 + {}_{10}E_{30} \times 8.6602$

${}_{10}E_{30} = (15.0364 - 8.7201)/8.6602 = 0.72935$

Actuarial present value (APV) of benefits =

$$= 1000 \times A^{1}_{40:\overline{10}|} + 2000 \times {}_{10}E_{30} \times A^{1}_{50:\overline{10}|}$$

$$= 16.66 + 2 \times 0.72935 \times 32.61 = 64.23$$

APV of premiums = $\pi \times \ddot{a}_{30:\overline{10}|} + 2\pi \times 0.72935 \times \ddot{a}_{40:\overline{10}|}$

$$= \pi \times 8.7201 + 2 \times \pi \times 0.72935 \times 8.6602$$

$$= 21.3527\pi$$

$$\pi = 64.23 / 21.3527 = 3.01$$

Question #222
Key: A

$${}_{15}V_{25} = P_{25}\,\ddot{s}_{25:\overline{15|}} - \frac{A^{1}_{25:\overline{15|}}}{{}_{15}E_{25}} \qquad \text{(this is the retrospective reserve calculation)}$$

$$P^{1}_{25:\overline{15|}} = P_{25:\overline{15|}} - P_{25:\overline{15|}}^{\;\;1} = 0.05332 - 0.05107$$
$$= 0.00225$$
$$= \frac{A^{1}_{25:\overline{15|}}}{\ddot{a}_{25:\overline{15|}}}$$

$$0.05107 = P_{25:\overline{15|}}^{\;\;1} = \frac{{}_{15}E_{25}}{\ddot{a}_{25:\overline{15|}}} = \frac{1}{\ddot{s}_{25:\overline{15|}}}$$

$$\frac{A^{1}_{25:\overline{15|}}}{{}_{15}E_{25}} = \frac{A^{1}_{25:\overline{15|}}/\ddot{a}_{25:\overline{15|}}}{{}_{15}E_{25}/\ddot{a}_{25:\overline{15|}}} = \frac{0.00225}{0.05107} = 0.04406$$

$$P_{25}\,\ddot{s}_{25:\overline{15|}} = \frac{0.01128}{0.05107} = 0.22087$$

$$25{,}000\,{}_{15}V_{25} = 25{,}000(0.22087 - 0.04406) = 25{,}000(0.17681) = 4420$$

There are other ways of getting to the answer, for example writing

A: the retrospective reserve formula for ${}_{15}V_{25}$.

B: the retrospective reserve formula for ${}_{15}V^{1}_{25:\overline{15|}}$, which = 0

Subtract B from A to get

$$\left(P_{25} - P^{1}_{25:\overline{15|}}\right)\ddot{s}_{25:\overline{15|}} = {}_{15}V_{25}$$

Question #223
Key: C

ILT:

We have $p_{70} = 6{,}396{,}609/6{,}616{,}155 = 0.96682$

${}_{2}p_{70} = 6{,}164{,}663/6{,}616{,}155 = 0.93176$

$e_{70:\overline{2|}} = 0.96682 + 0.93176 = 1.89858$

CF: 0.93176 $= {}_{2}p_{70} = e^{-2\mu} \;\Rightarrow\; \mu = 0.03534$

Hence $e_{70:\overline{2|}} = p_{70} + {}_{2}p_{71} = e^{-\mu} + e^{-2\mu} = 1.89704$

DM: Since l_{70} and ${}_{2}p_{70}$ for the DM model equal the ILT, therefore l_{72} for the DM model also equals the ILT. For DM we have $l_{70} - l_{71} = l_{71} - l_{72} \Rightarrow l_{71}^{(\mathrm{DM})} = 6,390,409$

Hence $e_{70:\overline{2}|} = 6,390,409/6,616,155 + 6,164,663/6,616,155 = 1.89763$

So the correct order is CF < DM < ILT
You could also work with *p*'s instead of *l*'s. For example, with the ILT,

$$p_{70} = (1 - 0.03318) = 0.96682$$
$${}_2 p_{70} = (0.96682)(1 - 0.03626) = 0.93176$$

Note also, since $e_{70:\overline{2}|} = p_{70} + {}_2 p_{70}$, and ${}_2 p_{70}$ is the same for all three, you could just order p_{70}.

Question #224
Key: D

$$l_{60}^{(\tau)} = 1000$$
$$l_{61}^{(\tau)} = 1000(0.99)(0.97)(0.90) = 864.27$$
$$d_{60}^{(\tau)} = 1000 - 864.27 = 135.73$$
$$d_{60}^{(3)} = 135.73 \times \frac{-\ln(0.9)}{-\ln\left[(0.99)(0.97)(0.9)\right]} = \frac{0.1054}{0.1459} = 98.05$$
$$l_{62}^{(\tau)} = 864.27(0.987)(0.95)(0.80) = 648.31$$
$$d_{61}^{(\tau)} = 864.27 - 648.31 = 215.96$$
$$d_{61}^{(3)} = 215.96 \times \frac{-\ln(0.80)}{-\ln\left[(0.987)(0.95)(0.80)\right]} = \frac{0.2231}{0.2875} = 167.58$$

So $d_{60}^{(3)} + d_{61}^{(3)} = 98.05 + 167.58 = 265.63$

Question #225
Key: B

$${}_t p_{40} = e^{-0.05t}$$
$${}_t p_{50} = (60 - t)/60$$
$$\mu_{50+t} = 1/(60 - t)$$

$$\int_0^{10} {}_t p_{40:50}\,\mu_{50+t}\,dt = \int_0^{10} \frac{e^{-0.05t}}{60}\,dt = -\frac{1}{60}\,\frac{e^{-0.05t}}{(0.05)}\Bigg|_0^{10}$$

$$= \frac{20}{60}\left(1 - e^{-0.5}\right) = 0.13115$$

Question #226
Key: A

Actual payment (in millions) $= \frac{3}{1.1} + \frac{5}{1.1^2} = 6.860$

$q_3 = 1 - \frac{0.30}{0.60} = 0.5$

${}_{1|}q_3 = \frac{0.30 - 0.10}{0.60} = 0.333$

Expected payment $= 10\left(\frac{0.5}{1.1} + \frac{0.333}{1.1^2}\right) = 7.298$

Ratio $\frac{6.860}{7.298} = 94\%$

Question #227
Key: E

At duration 1

$K(x)$	${}_1L$	Prob
1	$v - P^1_{x:\overline{2}\rvert}$	q_{x+1}
>1	$0 - P^1_{x:\overline{2}\rvert}$	$1 - q_{x+1}$

So $Var\left({}_1L\right) = v^2 q_{x+1}\left(1 - q_{x+1}\right) = 0.1296$

That really short formula takes advantage of
$Var\left(aX + b\right) = a^2 Var(X)$, if *a* and *b* are constants.
Here *a* = *v*; $b = P^1_{x:\overline{2}\rvert}$; *X* is binomial with $p\left(X = 1\right) = q_{x+1}$.

Alternatively, that same formula for *Var* arises from Hattendorf, since

${}_2V = 0$ and $Var\left({}_2L\right) = 0$

Alternatively, evaluate $P^{1}_{x:\overline{2|}} = 0.1303$

${}_1L = 0.9 - 0.1303 = 0.7697$ if $K(x) = 1$

${}_1L = 0 - 0.1303 = -0.1303$ if $K(X) > 1$

$E\left({}_1L\right) = (0.2)(0.7697) + (0.8)(-0.1303) = 0.0497$

$E\left({}_1L^2\right) = (0.2)(0.7697)^2 + (0.8)(-0.1303)^2 = 0.1320$

$Var\left({}_1L\right) = 0.1320 - (0.0497)^2 = 0.1295$

Question #228
Key: C

$$\bar{P}\left(\bar{A}_x\right) = \frac{\bar{A}_x}{\bar{a}_x} = \frac{\bar{A}_x}{\left(\dfrac{1-\bar{A}_x}{\delta}\right)} = \frac{\delta \bar{A}_x}{1-\bar{A}_x} = \frac{(0.1)\left(\frac{1}{3}\right)}{\left(1-\frac{1}{3}\right)} = 0.05$$

$$Var(L) = \left(1 + \frac{\bar{P}\left(\bar{A}_x\right)}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\frac{1}{5} = \left(1 + \frac{0.05}{0.10}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\left({}^2\bar{A}_x - \bar{A}_x^2\right) = 0.08888$$

$$Var[L'] = \left(1 + \frac{\pi}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\frac{16}{45} = \left(1 + \frac{\pi}{0.1}\right)^2 (0.08888)$$

$$\left(1 + \frac{\pi}{0.1}\right)^2 = 4$$

$$\pi = 0.1$$

Question #229
Key: E

Seek g such that $\Pr\left\{\bar{a}_{\overline{T(40)|}} > g\right\} = 0.25$

$\bar{a}_{\overline{T|}}$ is a strictly increasing function of T.

$\Pr\{T(40) > 60\} = 0.25$ since ${}_{60}p_{40} = \dfrac{100-40}{120-40} = 0.25$

$\therefore \Pr\left\{\bar{a}_{\overline{T(40)}|} > \bar{a}_{\overline{60}|}\right\} = 0.25$

$g = \bar{a}_{\overline{60}|} = 19.00$

Question 230
Key: B

$A_{51:\overline{9}|} = 1 - d\ddot{a}_{51:\overline{9}|} = 1 - \left(\dfrac{0.05}{1.05}\right)(7.1) = 0.6619$

${}_{11}V = (2000)(0.6619) - (100)(7.1) = 613.80$

$\left({}_{10}V + P\right)(1.05) = {}_{11}V + q_{50}\left(2000 - {}_{11}V\right)$

$\left({}_{10}V + 100\right)(1.05) = 613.80 + (0.011)(2000 - 613.80)$

${}_{10}V = 499.09$

where $q_{50} = (0.001)(10) + (0.001) = 0.011$

Alternatively, you could have used recursion to calculate $A_{50:\overline{10}|}$ from $A_{51:\overline{9}|}$, then $\ddot{a}_{50:\overline{10}|}$ from $A_{50:\overline{10}|}$, and used the prospective reserve formula for ${}_{10}V$.

Question #231
Key: C

$1000A_{81} = (1000A_{80})(1+i) - q_{80}(1000 - A_{81})$

$689.52 = (679.80)(1.06) - q_{80}(1000 - 689.52)$

$q_{80} = \dfrac{720.59 - 689.52}{310.48} = 0.10$

$$q_{[80]} = 0.5q_{80} = 0.05$$

$$1000A_{[80]} = 1000vq_{[80]} + vp_{[80]}\,1000A_{81}$$
$$= 1000 \times \frac{0.05}{1.06} + 689.52 \times \frac{0.95}{1.06} = 665.14$$

Question #232
Key: D

	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
42	776	8	16
43	752	8	16

$l_{42}^{(\tau)}$ and $l_{43}^{(\tau)}$ came from $l_{x+1}^{(\tau)} = l_x^{(\tau)} - d_x^{(1)} - d_x^{(2)}$

$$APV \text{ Benefits} = \frac{2000\left(8v + 8v^2\right) + 1000\left(16v + 16v^2\right)}{776} = 76.40$$

$$APV \text{ Premiums} = 34\left(\frac{776 + 752v}{776}\right) = (34)(1.92) = 65.28$$

$${}_2V = 76.40 - 65.28 = 11.12$$

Question #233
Key: B

$$p_{xx} = 1 - q_{xx} = 0.96$$

$$p_x = \sqrt{0.96} = 0.9798$$

$$p_{x+1:x+1} = 1 - q_{x+1:x+1} = 0.99$$

$$p_{x+1} = \sqrt{0.99} = 0.995$$

$$\ddot{a}_x = 1 + vp_x + v^2 \times {}_2p_x = 1 + \frac{0.9798}{1.05} + \frac{(0.9798)(0.995)}{1.05^2}$$
$$= 2.8174$$

$$\ddot{a}_{xx} = 1 + vp_{xx} + v^2 \times {}_2p_{xx} = 1 + \frac{0.96}{1.05} + \frac{(0.96)(0.99)}{1.05^2} = 2.7763$$

APV = $2000\,\ddot{a}_x + 2000\,\ddot{a}_x + 6000\,\ddot{a}_{xx}$

$$= (4000)(2.8174) + (6000)(2.7763)$$
$$= 27{,}927$$

Notes: The solution assumes that the future lifetimes are identically distributed. The precise description of the benefit would be a special 3-year temporary life annuity-due.

Question #234
Key: B

$${}_tp_x'^{(1)}\mu_x^{(1)}(t) = q_x'^{(1)} = 0.20$$
$${}_tp_x'^{(2)} = 1 - tq_x'^{(2)} = 1 - 0.08t$$
$${}_tp_x'^{(3)} = 1 - tq_x'^{(3)} = 1 - 0.125t$$
$$q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)}\mu_x^{(1)}(t)\,dt = \int_0^1 {}_tp_x'^{(2)}\,{}_tp_x'^{(3)}\,{}_tp_x'^{(1)}\mu_x^{(1)}(t)\,dt$$
$$= \int_0^1 (1 - 0.08t)(1 - 0.125t)(0.20)\,dt$$
$$= 0.2\int_0^1 \left(1 - 0.205t + 0.01t^2\right)dt$$
$$= 0.2\left[t - \frac{0.205t^2}{2} + \frac{0.01t^3}{3}\right]_0^1$$
$$= (0.2)\left[1 - 0.1025 + \frac{0.01}{3}\right] = 0.1802$$

Question #235
Key: B

$${}_1V_{40} = 1 - \frac{\ddot{a}_{41}}{\ddot{a}_{40}} = 1 - \frac{14.6864}{14.8166} = 0.00879$$
$${}_1CV_{40} = \frac{(1000)(1)}{3}(0.00879) = 2.93$$

$$ {}_1AS = \frac{\left(G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{40}^{(d)} - {}_1CV_{40} \times q_{40}^{(w)}}{1 - q_{40}^{(d)} - q_{40}^{(w)}} $$

$$ = \frac{(0.9G - 1.50)(1.06) - (1000)(0.00278) - (2.93)(0.2)}{1 - 0.00278 - 0.2} $$

$$ = \frac{0.954G - 1.59 - 2.78 - 0.59}{0.79722} $$

$$ = 1.197G - 6.22 $$

$$ {}_2AS = \frac{\left({}_1AS + G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{41}^{(d)} - {}_2CV_{40} \times q_{41}^{(w)}}{1 - q_{41}^{(d)} - q_{41}^{(w)}} $$

$$ = \frac{(1.197G - 6.22 + G - 0.1G - 1.50)(1.06) - (1000)(0.00298) - {}_2CV_{40} \times 0}{1 - 0.00298 - 0} $$

$$ = \frac{(2.097G - 7.72)(1.06) - 2.98}{0.99702} $$

$$ = 2.229G - 11.20 $$

$$ 2.229G - 11.20 = 24 $$

$$ G = 15.8 $$

Question #236
Key: A

$$ {}_5AS = \frac{\left({}_4AS + G(1 - c_4) - e_4\right)(1+i) - 1000q_{x+4}^{(1)} - {}_5CV \times q_{x+4}^{(2)}}{1 - q_{x+4}^{(1)} - q_{x+4}^{(2)}} $$

$$ = \frac{\left(396.63 + 281.77(1 - 0.05) - 7\right)(1+i) - 90 - 572.12 \times 0.26}{1 - 0.09 - 0.26} $$

$$ = \frac{(657.31)(1+i) - 90 - 148.75}{0.65} $$

$$ = 694.50 $$

$$(657.31)(1+i) = 90 + 148.75 + (0.65)(694.50)$$

$$1+i = \frac{690.18}{657.31} = 1.05$$

$$i = 0.05$$

Question #237
Key: C

Excluding per policy expenses, policy fee, and expenses associated with policy fee.
APV (actuarial present value) of benefits $= 25,000\,\bar{A}_{x:\overline{20|}} = (25,000)(0.4058) = 10,145$

Let *G* denote the expense-loaded premium, excluding policy fee.

$$\begin{aligned}\text{APV of expenses} &= (0.25-0.05)G + 0.05G\,\ddot{a}_{x:\overline{20|}} + \left[(2.00-0.50) + 0.50\,\ddot{a}_{x:\overline{20|}}\right](25,000/1000) \\ &= \left[0.20 + (0.05)(12.522)\right]G + \left[1.50 + (0.50)(12.522)\right]25 \\ &= 0.8261G + 194.025\end{aligned}$$

APV of premiums = $G\,\ddot{a}_{x:\overline{20|}} = 12.522G$

Equivalence principle:

APV premium = APV benefits + APV expenses
$$12.522G = 10,145 + 0.8261G + 194.025$$

$$G = \frac{10,339.025}{(12.522-0.8261)} = 883.99$$

This *G* is the premium excluding policy fee.

Now consider only year 1 per policy expenses, the year one policy fee (call it F_1), and expenses associated with F_1.

APV benefits = 0
APV premium = F_1

Equivalence principle

$$F_1 = 15 + 0.25F_1$$

$$F_1 = \frac{15}{0.75} = 20$$

Total year one premium = $G + F_1$
= 884+20
= 904

Question #238
Key: B

Get G as in problem 3; G = 884

Now consider renewal per policy expenses, renewal policy fees (here called F_R) and expenses associated with F_R.

APV benefits = 0

APV expenses = $\left(3+0.05F_R\right)a_{x:\overline{19|}}$
$= \left(3+0.05F_R\right)\left(12.522-1\right)$
$= 34.566+0.5761F_R$

APV premiums = $a_{x:\overline{19|}}F_R$
$= \left(12.522-i\right)F_R$
$= 11.522F_R$

Equivalence principle:

$$11.522F_R = 34.566+0.5761F_R$$
$$F_R = \frac{34.566}{11.522-0.5761} = 3.158$$

Total renewal premium = $G + F_R$
$= 884+3.16$
= 887

Since all the renewal expenses are level, you could reason that at the start of every renewal year, you collect F_R and pay expenses of $3+0.05F_R$, thus $F_R = \dfrac{3}{1-0.05} = 3.16$

Such reasoning is valid, but only in the case the policy fee and all expenses in the policy fee calculation are level.

Question #239
Key: B

Let P denote the expense-loaded premium

From problem 3, APV of benefits = 10,145
From calculation exactly like problem 3,
APV of premiums = 12.522 P

APV of expenses = $(0.25-0.05)P+0.05P\,\ddot{a}_{x:\overline{20}|}+\left[(2.00-0.50)+0.50\,\ddot{a}_{x:\overline{20}|}\right](25000/1000)$

$$+(15-3)+3\ddot{a}_{x:\overline{20}|}$$
$$=0.20P+(0.05P)(12.522)+(1.50+(0.50)(12.522))(25)+12+(3)(12.522)$$
$$=0.8261P+243.59$$

Equivalence principle:

$$12.522\,P=10{,}145+0.8261P+244$$
$$P=\frac{10{,}389}{12.522-0.8261}$$
$$=888$$

Question #240
Key: D

Let G denote the expense-loaded premium.
Actuarial present value (APV) of benefits = $1000\bar{A}_{40:\overline{20}|}$

APV of premiums = $G\,\ddot{a}_{40:\overline{10}|}$

APV of expenses $=(0.04+0.25)G+10+(0.04+0.05)G\,a_{40:\overline{9}|}+5a_{40:\overline{19}|}$

$$=0.29G+10+0.09G\,a_{40:\overline{9}|}+5a_{40:\overline{19}|}$$
$$=0.2G+10+0.09G\,\ddot{a}_{40:\overline{10}|}+5a_{40:\overline{19}|}$$

(The above step is getting an $\ddot{a}_{40:\overline{10}|}$ term since all the answer choices have one. It could equally well have been done later on).

Equivalence principle:

$$G\,\ddot{a}_{40:\overline{10}|} = 1000\bar{A}_{40:\overline{20}|} + 0.2G + 10 + 0.09G\,\ddot{a}_{40:\overline{10}|} + 5\,a_{40:\overline{19}|}$$

$$G\left(\ddot{a}_{40:\overline{10}|} - 0.2 - 0.09\,\ddot{a}_{40:\overline{10}|}\right) = 1000\bar{A}_{40:\overline{20}|} + 10 + 5\,a_{40:\overline{19}|}$$

$$G = \frac{1000\bar{A}_{40:\overline{20}|} + 10 + 5\,a_{40:\overline{19}|}}{0.91\ddot{a}_{40:\overline{10}|} - 0.2}$$

Question #241
Key: C

Let G denote the expense-loaded premium excluding policy fee.
Actuarial Present Value (APV) of benefits $= 1000A_x$

$$= 100{,}000\left(1 - d\,\ddot{a}_x\right)$$

$$= 100{,}000\left(1 - \left(\frac{0.04}{1.04}\right)(10.8)\right)$$

$$= 58{,}462$$

APV of premiums $= G\,\ddot{a}_x = 10.8G$

Excluding per policy expenses and expenses on the policy fee,
APV(expenses) $= 0.5G + (2.0)(100) + \left(0.04G + (0.5)(100)\right)a_x$

$$= 0.5G + 200 + (0.04G + 50)(9.8)$$

$$= 0.892G + 690$$

Equivalence principle:

$$10.8G = 58{,}462 + 0.892G + 690$$

$$G = \frac{59{,}152}{9.908} = 5970.13$$

Let F denote the policy fee.
APV of benefits = 0
APV of premiums = $F\,\ddot{a}_x = 10.8F$
APV of expenses = $150 + 25a_x + 0.5F + 0.04F\,a_x$

$$= 150 + 25(9.8) + 0.5F + 0.04F(9.8)$$

$$= 395 + 0.892F$$

Equivalence principle:

$$10.8F = 395 + 0.892F$$

$$F = \frac{395}{10.8 - 0.892}$$

$$= 39.87$$

$$\text{Total premium } = G + F$$

$$= 5970.13 + 39.87$$

$$= 6010$$

Note: Because both the total expense-loaded premium and the policy fee are level, it was not necessary to calculate the policy fee separately. Let *P* be the combined expense-loaded premium.

APV benefits = 58,462
APV premiums = 10.8*P*
APV expenses = $0.892P + 690 + 150 + (25)(9.8)$
= $0.892P + 1085$

where 0.892*P* + 690 is comparable to the expenses in *G* above, now including all percent of premium expense.

Equivalence principle:

$$10.8P = 58,462 + 0.892P + 1085$$

$$P = \frac{59547}{10.8 - 0.892}$$

$$= 6010$$

This (not calculating the policy fee separately, even though there is one) only works with level premiums and level policy fees.

Question #242
Key: C

$$_{11}AS = \frac{\left(_{10}AS + G - c_{10}\,G - e_{10}\right)(1+i) - 10,000q_{x+10}^{(d)} - {}_{11}CV\,q_{x+10}^{(w)}}{1 - q_{x+10}^{(d)} - q_{x+10}^{(w)}}$$

$$= \frac{\left(1600 + 200 - (0.04)(200) - 70\right)(1.05) - (10,000)(0.02) - (1700)(0.18)}{1 - 0.02 - 0.18}$$

$$= \frac{1302.1}{0.8}$$

$$= 1627.63$$

Question #243
Key: E

Let *G* denote the expense-loaded premium.
G = benefit premium plus level premium (*e*) for expenses.
Expense reserve = Actuarial Present Value (APV) of future expenses – APV of future expense premiums.

At duration 9, there is only one future year's expenses and due future premium, both payable at the start of year 10.

Expense reserve = APV of expenses – APV of expense premiums

= 0.10*G* + 5 – e

$= 0.10\left(1000P_{35:\overline{10}|} + e\right) + 5 - e$

$= (0.10)(76.87) + 5 - 0.9e$

= 12.687 – 0.9e

12.687 – 0.9*e* = – 1.67

e = 15.95

$$G = 1000P_{35:\overline{10}|} + e$$
$$= 76.87 + 15.95$$
$$= 92.82$$

Question #244
Key: C

$$ {}_4AS = \frac{\left({}_3AS + G - c_4G - e_4\right)(1+i) - 1000q_{x+3}^{(d)} - {}_4CVq_{x+3}^{(w)}}{1 - q_{x+3}^{(d)} - q_{x+3}^{(w)}} $$

Plugging in the given values:

$$ {}_4AS = \frac{\left(25.22 + 30 - (0.02)(30) - 5\right)(1.05) - 1000(0.013) - 75(0.05)}{1 - 0.013 - 0.05} $$

$$ = \frac{35.351}{0.937} $$

= 37.73

With higher expenses and withdrawals:

$$ {}_4AS^{\text{ revised}} = \frac{25.22 + 30 - (1.2)\left((0.02)(30) + 5\right)(1.05) - 1000(0.013) - 75(1.2)(0.05)}{1 - 0.013 - (1.2)(0.05)} $$

$$ = \frac{(48.5)(1.05) - 13 - 4.5}{0.927} $$

$$ = \frac{33.425}{0.927} $$

= 36.06

$$ {}_4AS - {}_4AS^{\text{ revised}} = 37.73 - 36.06 $$

= 1.67

Question #245
Key: E

Let G denote the expense-loaded premium.
APV (actuarial present value) of benefits $= 1000\,{}_{10|20}A_{30}$.

APV of premiums $= G\ddot{a}_{30:\overline{5}|}$.

APV of expenses $= (0.05+0.25)G+20$ first year

$+\left[(0.05+0.10)G+10\right]a_{30:\overline{4}|}$ years 2-5

$+10\;{}_{5|}\ddot{a}_{35:\overline{4}|}$ years 6-10 (there is no premium)

$$= 0.30G + 0.15G\,a_{30:\overline{4}|} + 20 + 10\,a_{30:\overline{4}|} + 10\;{}_{5|}\ddot{a}_{30:\overline{5}|}$$

$$= 0.15G + 0.15G\ddot{a}_{30:\overline{5}|} + 20 + 10\,a_{30:\overline{9}|}$$

(The step above is motivated by the form of the answer. You could equally well put it that form later).

Equivalence principle:

$$G\ddot{a}_{30:\overline{5}|} = 1000\,{}_{10|20}A_{30} + 0.15G + 0.15G\,\ddot{a}_{30:\overline{5}|} + 20 + 10\,a_{30:\overline{9}|}$$

$$G = \frac{\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9}|}\right)}{(1-0.15)\ddot{a}_{30:\overline{5}|} - 0.15}$$

$$= \frac{\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9}|}\right)}{0.85\ddot{a}_{30:\overline{5}|} - 0.15}$$

Question #246
Key: E

Let G denote the expense-loaded premium
APV (actuarial present value) of benefits

$$=(0.1)(3000)v+(0.9)(0.2)(2000)v^2+(0.9)(0.8)1000v^2$$

$$=\frac{300}{1.04}+\frac{360}{1.04^2}+\frac{720}{1.04^2}=1286.98$$

APV of premium = G
APV of expenses $=0.02G+0.03G+15+(0.9)(2)v$

$$=0.05G+\frac{16.8}{1.04}$$

$$=0.05G+16.15$$

Equivalence principle: $G=1286.98+0.05G+16.15$

$$G=\frac{1303.13}{1-0.05}=1371.72$$

Question #247
Key: C

APV (actuarial present value) of benefits = 3499 (given)

APV of premiums $=G+(0.9)(G)v$

$$=G+\frac{0.9G}{1.05}=1.8571G$$

APV of expenses, except settlement expenses,

$$=\left[25+(4.5)(10)+0.2G\right]+(0.9)\left[10+(1.5)(10)+0.1G\right]v+(0.9)(0.85)\left[10+(1.5)(10)\right]v^2$$

$$=70+0.2G+\frac{0.9(25+0.1G)}{1.05}+\frac{0.765(25)}{1.05^2}$$

=108.78+0.2857G

Settlement expenses are $20+(1)(10)=30$, payable at the same time the death benefit is paid.

So APV of settlement expenses $=\left(\frac{30}{10,000}\right)$ APV of benefits

$$= (0.003)(3499)$$
$$= 10.50$$

Equivalence principle:

$$1.8571G = 3499 + 108.78 + 0.2857G + 10.50$$
$$G = \frac{3618.28}{1.8571 - 0.2857} = 2302.59$$

Question #248
Key: D

$$\ddot{a}_{50:\overline{20}|} = \ddot{a}_{50} - {}_{20}E_{50}\,\ddot{a}_{70}$$
$$= 13.2668 - (0.23047)(8.5693)$$
$$= 11.2918$$
$$A_{50:\overline{20}|} = 1 - d\,\ddot{a}_{50:\overline{20}|} = 1 - \left(\frac{0.06}{1.06}\right)(11.2918)$$
$$= 0.36084$$

Actuarial present value (APV) of benefits $= 10,000 A_{50:\overline{20}|}$
$= 3608.40$

APV of premiums $= 495\,\ddot{a}_{50:\overline{20}|}$
$= 5589.44$

APV of expenses $= (0.35)(495) + 20 + (15)(10) + \left[(0.05)(495) + 5 + (1.50)(10)\right] a_{50:\overline{19}|}$
$= 343.25 + (44.75)(11.2918 - 1)$
$= 803.81$

APV of amounts available for profit and contingencies
= APV premium – APV benefits – APV expenses
= 5589.44 – 3608.40 – 803.81
= 1177.23

Question #249.
Key: B

$$q^1_{xy} = \int_0^1 {}_tp_{xy}\,\mu_{x+t}\,dt$$

$$= \int_0^1 {}_tp_x\,{}_tp_y\,\mu_{x+t}\,dt \quad \text{due to independence}$$

$$= q_x \int_0^1 {}_tp_y\,dt$$

$$= q_x \int_0^1 e^{-0.25t}\,dt$$

$$= q_x \left(-\frac{e^{-0.25t}}{0.25}\right)\Big|_0^1$$

$$= q_x \left(\frac{1-e^{-0.25}}{0.25}\right) = 0.8848\,q_x = 0.125$$

$$q_x = 0.141$$

(B)

Question #250

This is the probability ${}_2Q_1^{(1,1)}$, which is just the $(1,1)$-entry of

$$Q_1 Q_2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.73333 & 0.26667 \\ 0.33333 & 0.66667 \end{bmatrix},$$

namely $0.75(0.73333) + 0.25(0.33333) = 0.63333$

Question #251

This can be computed as ${}_2Q_1^{(1,1)} Q_3^{(1,2)}$ ${}_2Q_1^{(1,1)}$ is just the $(1,1)$-entry of

$$Q_1 Q_2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.73333 & 0.26667 \\ 0.33333 & 0.66667 \end{bmatrix},$$

namely $0.75(0.73333) + 0.25(0.33333) = 0.63333$. So the answer is $(0.63333)(0.275) = 0.17417$

Question #252

This probability is just ${}_3P_2^{(2)} = Q_1^{(2,2)} Q_2^{(2,2)} Q_3^{(2,2)} = (0.7)(0.66667)(0.65) = 0.30333$

Question #253

The triple-product summation for this actuarial present value is $Q_3^{(2,1)}(10)(\frac{1}{1.15}) + Q_3^{(2,2)} Q_4^{(2,1)}[10(1.1)](\frac{1}{1.15^2}) = \frac{(0.35)(10)}{1.15} + \frac{(0.65)(0.36)[10(1.1)]}{1.15^2} = 4.9898$.

Question #254

The triple-product summation for this actuarial present value is

$$[Q_3^{(1,2)}Q_4^{(2,1)}][10(1\ 1)](\frac{1}{1\ 15^2}) = \frac{(0.275)(0.36)(11)}{1\ 15^2} = 0\ 82344$$

Question #255

You can compute $APV_{2@3}$, the actuarial present value of these cash flows as seen from State #2 at time 3, by splitting off the first time period from the remaining periods:

$$APV_{2@3} = Q_3^{(2,1)}\,{}_4C^{(2,1)}v + Q_3^{(2,1)}vAPV_{1@4} + Q_3^{(2,2)}\,{}_4C^{(2,2)}v + Q_3^{(2,2)}vAPV_{2@4},$$

which equals $(0.4)(4)(0.8) + (0\ 4)(0.8)(5) + (0.6)(5)(0\ 8) + (0.6)(0.8)(7) = 8.64$.

Question #256

The triple-product summation for this actuarial present value is

$$(1)(5)(1) + Q_3^{(2,2)}(5)v = 5 + \frac{(0.65)(5)}{1.15} = 7\ 8261$$

Question #257

The benefit reserve is the actuarial present value of future benefits minus that of future benefit premiums, as computed for a driver in State #2 at time 4 Since there is only one year of possible benefits and one certain premium, the benefit reserve is

$$Q_4^{(2,1)}[10(1.1)](\frac{1}{1.15}) - 3.1879 = \frac{(0.36)(11)}{1.15} - 3\ 1879 = 0.25558.$$

Question #258

The benefit reserve is the actuarial present value of future benefits minus that of future benefit premiums, as computed for a resident in State #1 at time 2. Since there is only one year of possible benefits and one certain and one possible premium, the benefit reserve is

$$Q_2^{(1,2)}(100)v - [17\ 97 + Q_2^{(1,1)}(17\ 97)v] = -6.28$$

Question #259

The benefit-premium can be computed as the actuarial present value of the benefits divided by the actuarial present value of premiums of 1 The triple-product summation for the actuarial present value of the benefits is $Q_3^{(2,1)}(10)(\frac{1}{1\ 15}) + Q_3^{(2,2)}Q_4^{(2,1)}[10(1\ 1)](\frac{1}{1\ 15^2}) = \frac{(0.35)(10)}{1\ 15} + \frac{(0.65)(0.36)[10(1.1)]}{1\ 15^2} = 4\ 9898$. That for the actuarial present value of premiums of 1 is

$$(1)(1)(1) + Q_3^{(2,2)}(1)v = 1 + \frac{(0.65)(1)}{1\ 15} = 1.5652$$

Thus the benefit premium is $\frac{4.9898}{1\ 5652} = 3\ 1879$.

Question #260

The benefit-premium can be computed as the actuarial present value of the benefits divided by the actuarial present value of premiums of 1. The triple-product summation for the actuarial present value of the benefits is

$$Q_0^{(2,2)}(100)v + Q_0^{(1,1)}Q_1^{(1,2)}(100)v^2 + {}_2Q_0^{(1,1)}Q_2^{(1,2)}(100)v^3,$$

which equals $(0.2)(100)(0.8)+(0.7)(0.3)(100)(0.64)+(0.35)(0.2)(100)(0.512) = 33.024$; here ${}_2Q_0^{(1,1)}$ was computed as the $(1,1)$-entry of $\boldsymbol{Q}_0\boldsymbol{Q}_1$, namely 0.35. The actuarial present value of premiums of 1 is

$$1 + Q_0^{(1,1)}v + {}_2Q_0^{(1,1)}v^2 + {}_3Q_0^{(1,1)}v^3,$$

which is $1+(0.7)(0.8)+(0.35)(0.64)+(0.105)(0.512) = 1.83776$; here ${}_3Q_0^{(1,1)}$ was computed as the $(1,1)$-entry of $\boldsymbol{Q}_0\boldsymbol{Q}_1\boldsymbol{Q}_2$, namely 0.105. Finally, the benefit premium is $\frac{33.024}{1.83776} = 17.970$

Question #261
Key: A

The insurance is payable on the death of (y), if (x) predeceases (y).

$$E[Z] = \bar{A}_{x\overset{2}{y}} = \int_0^\infty v^t \, {}_tq_x \, {}_tp_y \, \mu_{y+t} \, dt$$

$$= \int_0^\infty e^{-.06t}\left(1-e^{-.07t}\right)\left(e^{-.09t}\right)(.09)\, dt$$

$$= .09 \int_0^\infty \left(e^{-.15t} - e^{-.22t}\right) dt$$

$$= .09 \left(\frac{1}{.15} - \frac{1}{.22}\right)$$

$$= .191 \quad \text{(A)}$$

Question #262
Key: C

$${}_tp_x = \frac{95-x-t}{95-x} \qquad \mu_{x+t} = \frac{1}{95-x-t} \qquad {}_tp_y = e^{-\mu t}$$

$$\int_0^n {}_tp_{xy}\,\mu_{x+t}\,dt = \int_0^n \frac{e^{-\mu t}}{95-x}\,dt = \frac{1-e^{-\mu n}}{\mu(95-x)} \quad \text{(C)}$$

Question #263
Key: A

$$0.25\,q^{\,2}_{30.5\,:\,40.5} = \int_0^{0.25} {}_tp_{30.5}\,\mu_{30.5+t}\;{}_tq_{40.5}\,dt$$

$$= \int_0^{0.25} \frac{0.6t}{1-(0.5)(0.6)} \times \frac{0.4}{1-(0.5)(0.4)}\,dt$$

$$= \int_0^{0.25} \frac{0.24t}{0.56}\,dt$$

$$= \left(\frac{0.24}{0.56}\right)\frac{t^2}{2}\Big|_0^{0.25}$$

$$= 0.0134 \quad \text{(A)}$$

Question #264
Key: B

Under hyperbolic (Balducci),

$$\frac{1}{l_{[60]+0.8}} = 0.2\left(\frac{1}{l_{[60]}}\right) + 0.8\left(\frac{1}{l_{[60]+1}}\right)$$

$$= \frac{0.2}{80{,}625} + \frac{0.8}{79{,}954}$$

$$= 0.0000124864$$

$$l_{[60]+0.8} = 80{,}087$$

$$\frac{1}{l_{[60]+1.5}} = 0.5\left(\frac{1}{l_{[60]+1}}\right) + 0.5\left(\frac{1}{l_{[60]+2}}\right)$$

$$= \frac{0.5}{79{,}954} + \frac{0.5}{78{,}839}$$

$$= 0.0000125956$$

$$l_{[60]+1.5} = 79{,}393$$

$$1000\, {}_{0.7}q_{[60]+0.8} = 1000\left(1 - {}_{0.7}p_{[60]+0.8}\right)$$

$$= 1000\left(1 - \frac{l_{[60]+1.5}}{l_{[60]+0.8}}\right)$$

$$= 1000\left(1 - \frac{79{,}393}{80{,}087}\right)$$

$$= 8.67 \quad \text{(B)}$$

Question #265
Key: D

$$q^1_{xy} = \int_0^1 {}_tp_y \, {}_tp_x \, \mu_{x+t} \, dt$$

$${}_tp_y = e^{-\int_0^t s\,ds} = e^{-s^2/2 |_0^t} = e^{-t^2/2}$$

$${}_tp_x = e^{-\int_0^t 5s\,ds} = e^{-\frac{5s^2}{2} |_0^t} = e^{-\frac{5t^2}{2}}$$

$$q^1_{xy} = \int_0^1 e^{-t^2/2} e^{-\frac{5t^2}{2}} \cdot 5t \, dt$$

$$= \int_0^1 e^{-3t^2} \cdot 5t \, dt$$

$$= \frac{5}{6} \int_0^1 6te^{-3t^2} \, dt$$

$$- 5/6 \; e^{-3t^2} |_0^1$$

$$5/6 \left[1 - e^{-3}\right]$$

$$= 0.7918$$ (D)

Question #266
Key: B

$$G = \int_0^5 \frac{t}{25} \cdot \frac{30-t}{30} \, \frac{1}{30-t} \, dt = \frac{t^2}{2 \cdot 25 \cdot 30} \Big|_0^5 = \frac{1}{60}$$

$$H = \frac{110-80-5}{110-80} \cdot \frac{110-85-5}{110-85} - \frac{110-80-10}{110-80} \cdot \frac{110-85-10}{110-85}$$

$$= \frac{25}{30} \cdot \frac{20}{25} - \frac{20}{30} \cdot \frac{15}{25} = \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$

$$G + H = \frac{1}{60} + \frac{16}{60} = \frac{17}{60} = .28 \quad \text{(B)}$$

Question #267
Key: D

$$s(x) = e^{-\int_0^x \mu_t \, dt}$$

$$= e^{-\int_0^x (80-t)^{-1/2} dt}$$

$$= e^{2(80-t)^{1/2} \Big|_0^x}$$

$$= e^{2[(80-x)^{1/2} - 80^{1/2}]}$$

$$F = s(10.5) = e^{2[69.5^{1/2} - 80^{1/2}]}$$

$$= e^{-2(0.60761)} = 0.29665$$

$$\text{Similarly, } s(10) = e^{2[70^{1/2} - 80^{1/2}]}$$

$$= e^{-2(0.57767)} = 0.31495$$

$$s(11) = e^{2[69^{1/2} - 80^{1/2}]}$$

$$= e^{-2(0.63765)} = 0.27935$$

With the hyperbolic / Balducci assumption:

$$\frac{1}{s(10.5)} = 0.5\left(\frac{1}{s(10)}\right) + 0.5\left(\frac{1}{s(11)}\right)$$

$$= \frac{0.5}{0.31495} + \frac{0.5}{0.27935} = 3.37742$$

$$G = s(10.5) = \frac{1}{3.37742} = 0.29608$$

$$F - G = 0.29665 - 0.29608 = 0.00057$$

(D)

Question #268
Key: A

$$E[Z] = 500\int_0^4 v^t \, {}_tp_{80}^{\beta} \cdot {}_tp_{80}^{s} \cdot \mu_{80+t}^{\beta} \cdot dt + 1000 \cdot \int_0^4 v^t \, {}_tp_{80}^{s} \, \mu_{80+t}^{s} \, {}_tq_{80}^{\beta} \cdot dt$$

$$= 500\int_0^4 \frac{(5-t)}{5} \cdot \frac{(4-t)}{4} \cdot \frac{1}{(5-t)} \cdot dt + 1000\int_0^4 \frac{4-t}{4} \cdot \frac{1}{4-t} \cdot \frac{t}{5} \cdot dt$$

$$= \frac{500}{20}\int_0^4 (4-t) \cdot dt + \frac{1000}{20}\int_0^4 t \cdot dt$$

$$= 25\,(16-8) + 50 \cdot 8 = 600 \quad \text{(A)}$$

Question #269
Key: A

$${}_{10}p_{\overline{30:50}} = {}_{10}p_{30} + {}_{10}p_{50} - {}_{10}p_{30:50}$$

$$= e^{-10(.05)} + e^{-10(.05)} - e^{-20(.05)}$$

$$= .845$$

$${}_{10}q_{\overline{30:50}} = 1 - .845 = .155 \quad \text{(A)}$$

Question #270
Key: C

$$\mathring{e}_{xy} = \int_0^\infty {}_tp_{xy}\,dt = \int_0^\infty e^{-.1t}\,dt$$

$$= \left. \frac{e^{-.1t}}{-.1} \right|_0^\infty$$

$$= 10$$

$$\mathring{e}_x = \mathring{e}_y = \int_0^\infty {}_tp_{x}\,dt = \int_0^\infty e^{-.05t}\,dt$$

$$= 20$$

$$\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy} = 30 \quad \text{(C)}$$

Question #271
Key: B

$$\bar{A}^{\,1}_{30:50} = \int_0^\infty e^{-\delta t} e^{-2\mu t} \mu\,dt = \frac{\mu}{2\mu + \delta} = \frac{.05}{.13} = .38 \quad \text{(B)}$$

Question #272
Key: B

$$Var[T(30:50)] = 2\int_0^\infty t e^{-.1t} dt - (\mathring{e}_{xy})^2$$

$$= 2\left(\frac{1}{2\mu}\right)^2 - \left(\frac{1}{2\mu}\right)^2$$

$$= \frac{1}{4\mu^2}$$

$$= 100$$

(B)

Question #273
Key: D

$$Cov = (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$$

$$= (20-10)(20-10)$$

$$= 100$$

(D)

Question #274
Key: E

$$({}_2V + \pi)(1+i) - q_{x+2}(\text{Benefit} - {}_3V) = {}_3V$$

$$(84 + 18)(1.07) - q_{x+2}(240 - 96) = 96$$

$$q_{x+2} = (109.14 - 96)/144$$

$$= 0.091 \quad \text{(E)}$$

Question #275
Key: A

$${}_4V = \frac{({}_3V + \pi)(1+i) - (q_{x+3})(\text{Benefit})}{p_{x+3}}$$

$$= \frac{(96+24)(1.06) - (0.101)(360)}{1 - 0.101}$$

$$= (127.2 - 36.36)/0.899$$

$$= 101.05 \quad \text{(A)}$$

Question #276
Key: C

Under Balducci/hyperbolic

$${}_{0.5}q_{x+3.5} = (0.5)\,q_{x+3.5} = (0.5)(0.101)$$

$$= 0.0505 \quad \text{(C)}$$

Question #277
Key: E

$${}_{3.5}V = v^{1/2}\left({}_{0.5}p_{x+3.5}\right){}_4V + v^{1/2}\left({}_{0.5}q_{x+3.5}\right)(\text{Benefit})$$

$$= \left(\frac{1}{1.06^{1/2}}\right)(1 - 0.0505)(101.05) + \left(\frac{1}{1.06^{1/2}}\right)(0.0505)(360)$$

$$= 110.85 \quad \text{(E)}$$

Question #278
Key: D

$$1 - {}_{10}p_{30:40} = 1 - {}_{10}p_{30} \cdot {}_{10}p_{40}$$
$$= 1 - \left(1 - \frac{10}{70}\right)\left(1 - \frac{10}{60}\right) = 1 - \frac{6}{7} \cdot \frac{5}{6} = \frac{2}{7} \quad \text{(D)}$$

Question #279
Key: A

$${}_{10}q^{2}_{30:40} = \int_0^{10} (1 - {}_tp_{40})\, {}_tp_{30}\, \mu_{30+t}\, dt$$
$$= \int_0^{10} \frac{1}{70} \; \frac{t}{60} \; dt$$
$$= \frac{1}{70} \; \frac{1}{60} \; \frac{t^2}{2} \Big|_0^{10} = \frac{1}{7 \cdot 6 \cdot 2} = 0.012 \quad \text{(A)}$$

Question #280
Key: A

$${}_{10|10}q_{\overline{30\,40}} = {}_{10|10}q_{30} + {}_{10|10}q_{40} - {}_{10|10}q_{30\,40}$$
$$= {}_{10}p_{30}\; {}_{10}q_{40} + {}_{10}p_{40}\; {}_{10}q_{50} - {}_{10}p_{30:40} \cdot {}_{10}q_{40:50}$$
$$= \left(1 - \frac{10}{70}\right)\left(\frac{10}{60}\right) + \left(1 - \frac{1}{6}\right)\left(\frac{1}{5}\right) - \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{6}\right)\left[1 - \left(1 - \frac{1}{6}\right)\left(1 - \frac{1}{5}\right)\right]$$
$$= \frac{1}{7} + \frac{1}{6} - \frac{5}{7} \cdot \frac{1}{3} = \frac{6 + 7 - 10}{42} = \frac{3}{42} = \frac{1}{14} \quad \text{(A)}$$
$$= 0.071$$

Question #281
Key: C

$$140{,}000 \int_0^{30} {}_tp_{30}\, \mu_{30+t}\, {}_tp_{40}\, dt + 180{,}000 \int_0^{30} {}_tp_{40}\, \mu_{40+t}\, {}_tp_{30}\, dt$$

$$000 \times \left\{ 140 \cdot \frac{1}{70} \int_0^{30} \left(1 - \frac{t}{60}\right) dt + 180 \cdot \frac{1}{60} \int_0^{30} \left(1 - \frac{t}{70}\right) dt \right\}$$

$$= 1000 \left[2 \left(t - \frac{t^2}{120}\right)\Big|_0^{30} + 3 \left(t - \frac{t^2}{140}\right)\Big|_0^{30} \right]$$

$$= 1000 \left[2 \left(30 - \frac{30}{4}\right) + 3 \left(30 - \frac{90}{14}\right) \right] = 115{,}714$$

(C)

Question #282
Key: B

$$PVFP = \bar{P} \int_0^{20} v^t\, {}_tp_{30}\, {}_tp_{40}\, dt$$

$$= \bar{P} \int_0^{20} 1 \cdot \left(1 - \frac{t}{70}\right)\left(1 - \frac{t}{60}\right) dt$$

$$= \bar{P} \int_0^{20} \left(1 - \frac{t}{70} - \frac{t}{60} + \frac{t^2}{4200}\right) dt$$

$$= \bar{P} \left(20 - \frac{400}{140} - \frac{400}{120} + \frac{8000}{3(4200)}\right)$$

$$= 14.4\, \bar{P}$$

(B)