Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.
Section 6.10. Premiums found including expenses.

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Premiums found including expenses.

When finding the annual premium expenses and commissions have to be taken into in account. Possible costs are underwriting (making the policy) and maintaining the policy. The annual premium which an insurance company charges is called the gross annual premium **gross annual premium**. The gross annual premium is called the contract premium **contract premium** and the loaded premium **loaded premium**. Usual, expenses are;

1. issue cost.
2. fixed amount and/or percentage of benefit premium.
3. (maintenance and other) fixed amount per policy.
4. fixed amount and/or percentage of contract amount.
5. settlement cost.

Often the expenses related with the contract amount, are given as per thousand expenses, i.e. the per thousand expenses are the expenses made for each $1,000 of the face value of the insurance.

For example, suppose that we have a whole life insurance on (x), with benefit payment made at the end of the year of death. Let b be the death benefit. Suppose that there is a fixed contract expense of c_1 paid at the beginning of year k. There is a expense paid at the beginning of year k, which is an r proportion of benefit premium. The issue cost is c_0. The settlement cost is s. Let G be the expense–augmented premium using the equivalence principle. We have that

$$G\ddot{a}_x = bA_x + rG\ddot{a}_x + c_1\ddot{a}_x + sA_x + c_0 = c_0 + (b+s)A_x + (rG + c_1)\ddot{a}_x.$$

So,

$$G = \frac{c_0 + (b+s)A_x + c_1\ddot{a}_x}{(1-r)\ddot{a}_x}.$$

Using that $P_x = \frac{A_x}{\ddot{a}_x}$ and $P_x + d = \frac{1}{\ddot{a}_x}$, we get that the expense–augmented annual benefit premium using the equivalence principle is

$$\begin{aligned} G &= \frac{c_0}{1-r}\frac{1}{\ddot{a}_x} + \frac{b+s}{1-r}P_x + \frac{c_1}{1-r} \\ &= \frac{c_0}{1-r}(P_x + d) + \frac{b+s}{1-r}P_x + \frac{c_1}{1-r} \\ &= \left(\frac{c_0 + b + s}{1-r}\right) P_x + \frac{c_0 d + c_1}{1-r}. \end{aligned}$$

The expense–augmented loss at issue random variable is

$$
\begin{aligned}
&{}_0L_e = bZ_x + rG\ddot{Y}_x + c_1\ddot{Y}_x + sZ_x + c_0 - G\ddot{Y}_x \\
=&c_0 + (b+s)Z_x - ((1-r)G - c_1)\ddot{Y}_x.
\end{aligned}
$$

When G is found using the equivalence principle,

$$\begin{aligned}
{}_0L_e &= c_0 + (b+s)Z_x - ((1-r)G - c_1)\ddot{Y}_x \\
&= c_0 + (b+s)Z_x - ((c_0+b+s)P_x + c_0 d)\ddot{Y}_x \\
&= c_0 + (b+s)Z_x - ((c_0+b+s)P_x + c_0 d)\frac{1-Z_x}{d} \\
&= -\frac{(c_0+b+s)P_x}{d} + (c_0+b+s)\left(1+\frac{P_x}{d}\right)Z_x \\
&= (c_0+b+s)\left(-\frac{P_x}{d} + \left(1+\frac{P_x}{d}\right)Z_x\right) \\
&= (c_0+b+s)L_x.
\end{aligned}$$

where $L_x = Z_x - P_x\ddot{Y}_x = -\frac{P_x}{d} + \left(1+\frac{P_x}{d}\right)Z_x$ is the loss without including expenses for a unit whole life insurance.
The variance of the expense–augmented loss is

$$\mathrm{Var}({}_0L_e) = (c_0+b+s)^2\mathrm{Var}(L_x) = (c_0+b+s)^2\left(1+\frac{P_x}{d}\right)^2\mathrm{Var}(Z_x).$$

Example 1

A whole life insurance policy with face value of $40,000 payable at the end of the year of death is made to (45). The following cost are incurred:
(i) 500 for making the contract.
(ii) 1% for each annual premium received.
(iii) Per policy expenses are 20 per year.
(iv) Per thousand expenses are $1.2 per year.
(v) 600 for settlement.
All expenses, except the settlement expense, are paid at the beginning of the year. $i = 4.5\%$ *and death is modeled using the De Moivre model with terminal age 95.*
(a) Calculate the gross annual premium using the equivalence principle.
(b) Calculate the expected–augmented loss for an insuree that dies 7 years, 5 months and 10 days after the issue of this policy.
(c) Calculate the variance of the expense–augmented loss.

Solution: (a) Using the equivalence principle,

$$
\begin{aligned}
& G\ddot{a}_{45} \\
=& (40000)A_{45} + 500 + G(0.01)\ddot{a}_{45} + 20\ddot{a}_{45} + (40)(1.2)\ddot{a}_{45} + (600)A_{45}, \\
=& (40600)A_{45} + 500 + G(0.01)\ddot{a}_{45} + (68)\ddot{a}_{45}.
\end{aligned}
$$

We have that

$$
\begin{aligned}
A_{45} &= \frac{a_{\overline{50}|}}{50} = 0.3952401556, \\
\ddot{a}_{45} &= \frac{1 - A_{45}}{d} = \frac{1 - 0.3952401556}{0.045/1.045} = 14.0438675.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
G =& \frac{500 + (40600)A_{45} + (68)\ddot{a}_{45}}{(0.99)\ddot{a}_{45}} \\
=& \frac{500 + (40600)(0.3952401556) + (68)(14.0438675)}{(0.99)(14.0438675)} \\
=& 1258.806984.
\end{aligned}
$$

Solution: (b) The loss is ($G = 1258.806984$)

$$\begin{aligned} L_e = &(40000)Z_{45} + 500 + G(0.01)\ddot{Y}_{45} + 20\ddot{Y}_{45} \\ &+ (40)(1.2)\ddot{Y}_{45} + (600)Z_{45} - G\ddot{Y}_{45}, \\ =&(40600)Z_{45} + 500 - 1064.926286\ddot{Y}_{45}. \end{aligned}$$

For an insuree that dies 7 years, 5 months and 10 days after the issue of this policy the expected–augmented loss is

$$L_e = (40600)v^8 + 500 - 1064.926286\ddot{a}_{\overline{8}|} = 21709.09774.$$

(c) We have that

$$^2A_{45} = \frac{a_{\overline{50}|0.045(2.045)}}{50} = 0.2146684865,$$

$$\mathrm{Var}({}_0L_e) = (c_0 + b + s)^2 \mathrm{Var}(L_x)$$

$$=(500 + 40000)^2 \frac{0.2146684865 - (0.3952401556)^2}{(1 - 0.3952401556)^2}$$

$$=262153827.8.$$

Fully continuous case

In the fully continuous case, the augmented–expense loss and the expense–augmented premium have expressions similar to the fully discrete cases. Let b be the death benefit death paid at the time of the death. Suppose that there is an annual rate of contract expenses of c_1 paid while (x) is alive. A proportion r of the benefit premium rate is paid in expenses. The issue cost is c_0. The settlement cost is s. Let G be the expense–augmented premium rate using the equivalence principle. We have that

$$G\overline{a}_x = b\overline{A}_x + rG\overline{a}_x + c_1\overline{a}_x + s\overline{A}_x + c_0 = c_0 + (b+s)\overline{A}_x + (rG + c_1)\overline{a}_x.$$

So,

$$G = \frac{c_0 + (b+s)\overline{A}_x + c_1\overline{a}_x}{(1-r)\overline{a}_x}.$$

The expense–augmented loss at issue random variable is

$${}_0\overline{L}_e = b\overline{Z}_x + rG\overline{Y}_x + c_1\overline{Y}_x + s\overline{Z}_x + c_0 - G\overline{Y}_x.$$

Using the equivalence principle

$$G = \left(\frac{c_0 + b + s}{1 - r}\right) \overline{P}_x + \frac{c_0\delta + c_1}{1 - r},$$

$${}_0\overline{L}_e = (c_0 + b + s)\overline{L}_x = (c_0 + b + s)(\overline{A}_x - \overline{P}_x \overline{a}_x)$$

and

$$\mathrm{Var}({}_0\overline{L}_e) = (c_0 + b + s)^2 \left(1 + \frac{\overline{P}_x}{\delta}\right)^2 \mathrm{Var}(\overline{Z}_x).$$

Example 2

For a fully continuous whole life insurance of \$50,000 on (x), *you are given:*

(i) The issuing expense is \$1,000.

(ii) The annual rate of continuous maintenance expense is 250.

(iii) There exists a continuous rate of expenses which is 0.10 of the benefit premium rate.

(iv) $\delta = 0.06$, $\overline{a}_x = 12$, $\mathrm{Var}(\overline{Z}_x) = 0.15$.

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(a) Calculate the expense–augmented annual premium rate using the equivalence principle.

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(a) Calculate the expense–augmented annual premium rate using the equivalence principle.

Solution: (a) We have that $\overline{A}_x = 1 - (0.06)(12) = 0.28$ and

$$Q(12) = Q\overline{a}_x = (50000)A_x + 1000 + 250\overline{a}_x + (0.10)Q\overline{a}_x$$
$$= (50000)(0.28) + 1000 + (250)(12) + 1.2Q = 18000 + 1.2Q$$

and $Q = \frac{18000}{12-1.2} = 1666.666667$.

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(b) Calculate $\mathrm{Var}({}_0L_e)$.

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(b) Calculate $\mathrm{Var}({}_0L_e)$.

Solution: (b) Using that $\mathrm{Var}({}_0\overline{L}_x) = \frac{{}^2\overline{A}_x - \overline{A}_x{}^2}{(\delta \overline{a}_x)^2}$, we get that

$$\mathrm{Var}({}_0\overline{L}_e) = (c_0 + b + s)^2 \mathrm{Var}({}_0\overline{L}_x) = (50000 + 1000)^2 \frac{0.15}{(0.06)^2(12)^2}$$

$=752604166.8.$

Different types of expenses can be paid during different length of times. This happens if benefits premiums and the policy are in hold for different periods.

Example 3

A 10–payment, fully discrete, 20–year term insurance policy with face value of 90000 payable at the time of death is made to (45). The following cost are incurred:
(i) 275 at the beginning of each year which the policy is active.
(ii) Per thousand expenses are $2.5 at the beginning of each year which the policy is active.
(iii) 1% for each annual premium received.
Assume that $i = 6\%$ *and death follows the life table for the USA population in 2004. Find the gross annual premium using the equivalence principle.*

Solution: Using the equivalence principle,

$$G\ddot{a}_{45:\overline{10}|} = (90000)A^{1}_{45:\overline{20}|} + 275\ddot{a}_{45:\overline{20}|} + (90)(2.5)\ddot{a}_{45:\overline{20}|}$$

$$+ G(0.01)\ddot{a}_{45:\overline{10}|} = (90000)A^{1}_{45:\overline{20}|} + 500\ddot{a}_{45:\overline{20}|} + G(0.01)\ddot{a}_{45:\overline{10}|},$$

$$G = \frac{(90000)A^{1}_{45:\overline{20}|} + 500\ddot{a}_{45:\overline{20}|}}{(0.99)\ddot{a}_{45:\overline{10}|}},$$

$$A^{1}_{45:\overline{20}|} = A_{45} - {}_{20}E_{45}A_{65} = 0.16656845 - (0.271632162)(0.37609614)$$

$$=0.06440864237,$$

$$\ddot{a}_{45:\overline{10}|} = \ddot{a}_{45} - {}_{10}E_{45}\ddot{a}_{55} = 14.723957 - (0.534696682)(13.160819)$$

$$=7.686910748,$$

$$\ddot{a}_{45:\overline{20}|} = \ddot{a}_{45} - {}_{20}E_{45}\ddot{a}_{65} = 14.723957 - (0.271632162)(11.022302)$$

$$=11.72994528,$$

$$G = \frac{(90000)(0.06440864237) + (500)(11.72994528)}{(0.99)(7.686910748)} = 1532.416116.$$

Often the first year expenses are different from the rest of the years. Usually, it is easier to express expenses as a level expense for all years plus an extra first year expense.

Example 4

For a 5–payment 20–year endowment insurance of $100,000 on (25), you are given:
(i) Percent of expense–loaded premium expenses are 10% in the first year and 2% thereafter.
(ii) Per active policy expenses are $200 in the first year and $80 in each year thereafter until death.
(iii) Expenses are paid at the beginning of each policy year.
(iv) Death benefits are payable at the end of the year of death.
(iv) The expense-loaded premium is determined using the equivalence principle.
(v) $i = 6\%$.
(vi) Mortality follows the life table for the USA population in 2004.
Calculate the expense-loaded premium using the equivalence principle.

Solution: Equating the APV of premiums and expenses, we get that

$$G\ddot{a}_{25:\overline{5}|} = (100000)\overline{A}_{25:\overline{20}|} + G(0.08) + G(0.02)\ddot{a}_{25:\overline{5}|} + 120 + 80\ddot{a}_{25:\overline{20}|}.$$

So,

$$G = \frac{(100000)A_{25:\overline{20}|} + 120 + 80\ddot{a}_{25:\overline{20}|}}{(0.98)\ddot{a}_{25:\overline{5}|} - (0.08)}.$$

We have that

$$A^1_{25:\overline{20}|} = A_{25} - {}_{20}E_{25}A_{45}$$
$$=0.065231113 - (0.302791379)(0.16656845) = 0.01479562233,$$
$$A_{25:\overline{20}|} = A^1_{25:\overline{20}|} + {}_{20}E_{25} = 0.01479562233 + 0.302791379$$
$$=0.3175870013,$$
$$\ddot{a}_{25:\overline{5}|} = \ddot{a}_{25} - {}_5E_{25}\ddot{a}_{30} = 16.51425 - (0.743683357)(16.212781)$$
$$=4.4570746,$$
$$\ddot{a}_{25:\overline{20}|} = \ddot{a}_{25} - {}_{20}E_{25}\ddot{a}_{45} = 16.51425 - (0.302791379)(14.723957)$$
$$=12.05596276,$$
$$G = \frac{(100000)A_{25:\overline{20}|} + 120 + 80\ddot{a}_{25:\overline{20}|}}{(0.98)\ddot{a}_{25:\overline{5}|} - (0.08)}$$
$$=\frac{(100000)(0.3175870013) + 120 + 80(12.05596276)}{(4.4570746)(0.98) - 0.08} = 7659.442515.$$

