Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.

Section 6.5. Premiums for insurance immediately paid with discrete premiums.

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In this section, we will consider the funding of insurance products paid at the time of death and funded at the beginning of the year.

Whole life insurance.

Suppose that an insurance company funds an immediately paid unity whole life insurance with benefit premiums P paid at the beginning of the year. The present value of the loss is

$$\overline{L}(A_x) = v^{T_x} - P\ddot{a}_{\overline{K_x}|} = \overline{Z}_x - P\ddot{Y}_x.$$

The actuarial present value of the loss is

$$E[\overline{L}(A_x)] = \overline{A}_x - P\ddot{a}_x.$$

If a whole life insurance is funded under the equivalence principle, then $E[\overline{L}(A_x)] = 0$. The benefit premium which satisfy the equivalence principle is denoted by $P(\overline{A}_x)$. We have that

$$P(\overline{A}_x) = \frac{\overline{A}_x}{\ddot{a}_x}.$$

If the funding scheme is limited to the first t years. The present value of the loss is

$$v^{T_x} - P\ddot{a}_{\overline{\min(K_x,t)}|} = \overline{Z}_x - P\ddot{Y}_{x:\overline{t}|}.$$

The present value of the loss is

$$\overline{A}_x - P\ddot{a}_{x:\overline{t}|}.$$

The benefit premium which satisfies the equivalence principle is

$${}_tP(\overline{A}_x) = \frac{\overline{A}_x}{\ddot{a}_{x:\overline{t}|}}.$$

n–year term insurance.

The present value of the loss for an immediately paid unity n–year term insurance with benefit premiums P paid at the beginning of the year is

$$v^{T_x} I(T_x \le n) - P\ddot{a}_{\overline{\min(K_x,n)}|} = \overline{Z}^{1}_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for this insurance contract is

$$\overline{A}^{1}_{x:\overline{n}|} - P\ddot{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$P(\overline{A}^{1}_{x:\overline{n}|}) = \frac{\overline{A}^{1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

The benefit premium which satisfies the equivalence principle for an immediately paid t–year funded n–year term insurance with benefit premiums P paid at the beginning of the year is

$$ {}_tP(\overline{A}^{\,1}_{x:\overline{n}|}) = \frac{\overline{A}^{\,1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{t}|}}. $$

n–year pure endowment.

The present value of the loss for an immediately paid unity n–year pure endowment funded at the beginning of the year is

$$v^n I(T_x > n) - P\ddot{a}_{\overline{\min(K_x,n)}|} = \overline{Z}_{x:\overline{n}|}^{\ \ 1} - P\ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for this insurance contract is

$$\overline{A}_{x:\overline{n}|}^{\ \ 1} - P\ddot{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$P(\overline{A}_{x:\overline{n}|}^{\ \ 1}) = \frac{\overline{A}_{x:\overline{n}|}^{\ \ 1}}{\ddot{a}_{x:\overline{n}|}}.$$

The present value of the loss of an immediately paid unity n–year pure endowment is funded with benefit premiums P at the beginning of the t years, where $1 \leq t \leq n$, is

$$v^n I(T_x > n) - P\bar{a}_{\overline{\min(K_x,t)}|} = \overline{Z}_{x:\overset{1}{\overline{n}|}} - P\overline{Y}_{x:\overline{t}|}.$$

The actuarial present value of the loss for this insurance contract is

$$\overline{A}_{x:\overset{1}{\overline{n}|}} - P\bar{a}_{x:\overline{t}|}.$$

The benefit premium which satisfies the equivalence principle is

$${}_tP(\overline{A}_{x:\overset{1}{\overline{n}|}}) = \frac{\overline{A}_{x:\overset{1}{\overline{n}|}}}{\bar{a}_{x:\overline{t}|}}.$$

n–year endowment.

The present value of the loss for an immediately paid unity n–year endowment funded with benefit premiums P paid at the beginning of the year is

$$v^{\min(n,T_x)} - P\ddot{a}_{\overline{\min(K_x,n)}|} = \overline{Z}_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for this insurance contract is

$$\overline{A}_{x:\overline{n}|} - P\ddot{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$P(\overline{A}_{x:\overline{n}|}) = \frac{\overline{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

The present value of the loss for a t–year funded n–year immediately paid endowment with benefit premiums P paid at the beginning of the year is

$$v^{\min(n,T_x)} - P\ddot{a}_{\overline{\min(K_x,n)}|} = \overline{Z}_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{t}|}.$$

The actuarial present value of the loss for a t–year funded n–year term insurance is

$$\overline{A}_{x:\overline{n}|} - P\ddot{a}_{x:\overline{t}|}.$$

The benefit premium which satisfies the equivalence principle is

$${}_tP(\overline{A}_{x:\overline{n}|}) = \frac{\overline{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{t}|}}.$$

n–year deferred insurance.

The present value of the loss for an immediately paid unity n–year deferred insurance funded with benefit premiums P paid at the beginning of the year is

$$v^{T_x} I(T_x > n) - P\ddot{a}_{\overline{K_x}|} = {}_n|\overline{Z}_x - P\ddot{Y}_x.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_n|\overline{A}_x - P\ddot{a}_x.$$

The benefit premium which satisfies the equivalence principle is

$$P({}_n|\overline{A}_x) = \frac{{}_n|\overline{A}_x}{\ddot{a}_x}.$$

