Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.

Section 6.6. Benefit premium for an *n*-year deferred annuity–due.

Extract from:
"Arcones' Manual for SOA Exam MLC. Spring 2009 Edition",
available at http://www.actexmadriver.com/

n–year deferred annuity due funded discretely.

The present value of the loss for a due n–year deferred contingent annuity contract funded at the beginning of the year with annual benefit payment of P is

$$v^n \ddot{a}_{\overline{K_x - n}|} I(K_x > n) - P \ddot{a}_{\overline{\min(K_x, n)}|} = {}_n|\ddot{Y}_x - P \ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_n|\ddot{a}_x - P \ddot{a}_{x:\overline{n}|}.$$

The annual benefit premium which satisfies the equivalence principle is

$$P({}_n|\ddot{a}_x) = \frac{{}_n|\ddot{a}_x}{\ddot{a}_{x:\overline{n}|}}.$$

Example 1

Jasmine is 45 years old and purchases a 20–year deferred contingent annuity with a face value of 40000 paid at the beginning of year while she is alive. This policy will be paid by level payments made at the beginning of the next 20 years while Jasmine is alive. Assume that $\delta = 0.05$ *and constant force of mortality* 0.02*. Find the annual benefit premium for this policy.*

Solution: We have that

$$
\begin{aligned}
{}_{20|}\ddot{a}_{45} &= \frac{e^{-(20)(0.07)}}{1 - e^{-0.07}} = 3.647550617,\\
\ddot{a}_{45:\overline{20}|} &= \frac{1 - e^{-(20)(0.07)}}{1 - e^{-0.07}} = 11.14399653,\\
(40000)\overline{P}({}_{20|}\ddot{a}_{45}) &= \frac{(40000)(3.647550617)}{11.14399653} = 13092.43271.
\end{aligned}
$$

The present value of the loss for an immediate n–year deferred contingent annuity contract funded at the beginning of each year in the deferred period while the individual is alive with an annual benefit payment of P is

$$v^n a_{\overline{K_x-n-1}|} I(K_x > n+1) - P\ddot{a}_{\overline{\min(K_x,n)}|} = {}_{n|}Y_x - P\ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss of this insurance product is

$$_{n|}a_x - P\ddot{a}_{x:\overline{n}|}.$$

The annual benefit premium which satisfies the equivalence principle is

$$P({}_{n|}a_x) = \frac{{}_{n|}a_x}{\ddot{a}_{x:\overline{n}|}}.$$

The present value of the loss for a continuous n–year deferred contingent annuity contract funded at the beginning of the year with a payment of P is

$$v^n \overline{a}_{\overline{T_x - n}|} I(T_x > n) - P \ddot{a}_{\overline{\min(K_x, n)}|} = {}_{n|}\overline{Y}_x - P \ddot{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_{n|}\overline{a}_x - P \ddot{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$P({}_{n|}\overline{a}_x) = \frac{{}_{n|}\overline{a}_x}{\ddot{a}_{x:\overline{n}|}}.$$

Example 2

Miguel is 50 years old and purchases a 15–year deferred contingent annuity with a face value of 50000 paid continuously while he is alive. This policy will be paid at the beginning of the year for the next 15 years while Miguel is alive. Assume that $i = 6.5\%$ and that mortatility is modeled using De Moivre's model with terminal age 90. Find the annual benefit premium for this policy.

Solution: We have that

$$\overline{a}_{\overline{25}|0.065} = \frac{1-(1.065)^{-25}}{\ln(1.065)} = 12.59014713,$$

$$\overline{A}_{65} = \frac{\overline{a}_{\overline{25}|0.065}}{25} = \frac{12.59014713}{25} = 0.5036058852,$$

$$\overline{a}_{65} = \frac{1-\overline{A}_{65}}{\delta} = \frac{1-0.5036058852}{\ln(1.065)} = 7.882424738,$$

$${}_{15|}\overline{a}_{50} = {}_{15}E_{50}\overline{a}_{65} = (1.065)^{-15}\frac{25}{40}(7.882424738) = 1.915559884,$$

$$A_{50:\overline{15}|} = \frac{a_{\overline{15}|0.065}}{40} + (1.065)^{-15}\frac{25}{40} = 0.4780832991,$$

$$\ddot{a}_{50:\overline{15}|} = \frac{1-0.4780832991}{(0.06/1.065)} = 8.551404407,$$

$$(50000)P({}_{15|}\overline{a}_{50}) = \frac{(50000)(1.915559884)}{8.551404407} = 11200.26485.$$

Example 3

Consider an annuity contract satisfying:
(i) it is funded with a net premium of P at the beginning of each year for n years while the individual is alive.
(ii) it makes a unity payment at the beginning of each year starting in n years while the individual is alive.
(iii) if the individual dies within n years, it returns the total accumulated premiums with interest.
Show that the annual premium for this contract is $P = \frac{\ddot{a}_{x+n}}{\ddot{s}_{\overline{n}|}}$.

Solution: We have that

$$P\ddot{a}_{x:\overline{n}|} = {}_{n}|\ddot{a}_x + P\sum_{k=1}^{n-1}\ddot{a}_{\overline{k}|}\mathbb{P}\{K_n = k\}$$

and

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=1}^{n-1}\ddot{a}_{\overline{k}|}\mathbb{P}\{K_n = k\} + \ddot{a}_{\overline{n}|}\mathbb{P}\{K_n \geq n\}.$$

Hence,

$$\sum_{k=1}^{n-1}\ddot{a}_{\overline{k}|}\mathbb{P}\{K_n = k\} = \ddot{a}_{x:\overline{n}|} - \ddot{a}_{\overline{n}|}\mathbb{P}\{K_n \geq n\} = \ddot{a}_{x:\overline{n}|} - \ddot{s}_{\overline{n}|}\cdot {}_{n}E_x.$$

Therefore,

$$P\ddot{a}_{x:\overline{n}|} = {}_{n}|\ddot{a}_x + P\left(\ddot{a}_{x:\overline{n}|} - \ddot{s}_{\overline{n}|}\cdot {}_{n}E_x\right),$$

and

$$P = \frac{{}_{n}|\ddot{a}_x}{\ddot{s}_{\overline{n}|}\cdot {}_{n}E_x} = \frac{\ddot{a}_{x+n}}{\ddot{s}_{\overline{n}|}}.$$

Notice that in the situation of the previous example, the insurer only retains the benefit premiums if the insuree survives n years. The APV of the retained benefit premiums is $P\ddot{a}_{\overline{n}|}\, {}_np_x$. Under the equivalence principle, $P\ddot{a}_{\overline{n}|} \cdot {}_np_x = {}_{n|}\ddot{a}_x$. Hence

$$P = \frac{{}_{n|}\ddot{a}_x}{\ddot{a}_{\overline{n}|} \cdot {}_np_x} = \frac{{}_{n|}\ddot{a}_x}{\ddot{s}_{\overline{n}|} \cdot {}_nE_x} = \frac{\ddot{a}_{x+n}}{\ddot{s}_{\overline{n}|}}.$$

Example 4

A special 20*–year deferred life insurance on* (50) *with face value 50000 is funded by annual benefit premiums at the beginning of the first* 20 *years while* (50) *is alive. If death happens during the deferral period, the insurer will return of the annual premiums with interest at the end of the year of death. Assume that* $i = 6\%$ *and death is modeled using the life table for the USA population in 2004. Calculate the amount of the benefit annual premium for this policy using the equivalence principle.*

Solution: Let P be the benefit annual premium. Using the equivalence principle,

$$P\ddot{a}_{50:\overline{20}|} = (50000)_{20|}\ddot{a}_{50} + P(\ddot{a}_{50:\overline{20}|} - \ddot{a}_{\overline{20}|} \cdot {}_{20}p_{50}).$$

Hence,

$$P = \frac{(50000)_{20|}\ddot{a}_{50}}{\ddot{a}_{\overline{20}|} \cdot {}_{20}p_{50}} = \frac{(50000)v^{20} \cdot {}_{20}p_{50} \cdot \ddot{a}_{50}}{\ddot{a}_{\overline{20}|} \cdot {}_{20}p_{50}} = \frac{(50000)\ddot{a}_{50}}{\ddot{s}_{\overline{20}|}}.$$

We have that

$$\ddot{a}_{50} = 14.0104, \ddot{s}_{\overline{20}|} = 38.99272668, P = \frac{(50000)(14.0104)}{38.99272668} = 17965.4017$$

n–year deferred annuity due funded continuously.

The present value of the loss for a due n–year deferred contingent annuity contract funded continuously during the deferred period while the individual is alive with a rate of benefit payments of P is

$$v^n \ddot{a}_{\overline{K_x - n}|} I(K_x > n) - P\bar{a}_{\overline{\min(T_x, n)}|} = {}_n|\ddot{Y}_x - P\overline{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_n|\ddot{a}_x - P\bar{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$\overline{P}({}_n|\ddot{a}_x) = \frac{{}_n|\ddot{a}_x}{\bar{a}_{x:\overline{n}|}}.$$

Example 5

Destiny is 35 years old and purchases a 30–year deferred contingent annuity with face value of 50000 paid at the end of year while she is alive. This policy will be paid by level continuous payments for the next 30 years while Destiny is alive. Assume that $\delta = 0.06$ *and constant force of mortality* 0.01*. Find the annual benefit premium for this policy.*

Solution:

$$
{}_{30|}\ddot{a}_{35} = \frac{e^{-(30)(0.07)}}{1 - e^{-0.07}} = 1.811320031,
$$

$$
\overline{a}_{35:\overline{30}|} = \frac{1 - e^{-(30)(0.07)}}{0.07} = 12.53633674,
$$

$$
(50000)\overline{P}({}_{30|}\ddot{a}_{35}) = \frac{(50000)(1.811320031)}{12.53633674} = 7224.279582.
$$

The present value of the loss for an immediate n–year deferred contingent annuity contract funded continuously with rate P is

$$v^n a_{\overline{K_x-n-1}|} I(K_x > n+1) - P\bar{a}_{\overline{\min(T_x,n)}|} = {}_n|Y_x - P\overline{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_n|a_x - P\bar{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$\overline{P}({}_n|a_x) = \frac{{}_n|a_x}{\bar{a}_{x:\overline{n}|}}.$$

The present value of the loss for a continuous n–year deferred contingent annuity contract funded continuously in the deferred period while the individual is alive with rate of benefit payment of P is

$$v^n \overline{a}_{\overline{T_x - n}|} I(T_x > n) - P\overline{a}_{\overline{\min(K_x, n)}|} = {}_n|\overline{Y}_x - P\overline{Y}_{x:\overline{n}|}.$$

The actuarial present value of the loss for a n–year term insurance is

$${}_n|\overline{a}_x - P\overline{a}_{x:\overline{n}|}.$$

The benefit premium which satisfies the equivalence principle is

$$\overline{P}({}_n|\overline{a}_x) = \frac{{}_n|\overline{a}_x}{\overline{a}_{x:\overline{n}|}}.$$

Example 6

Nicholas is 40 years old and purchases a 25*–year deferred continuous contingent annuity with face value of 60000. This policy will be paid by level continuous payments for the next 30 years while Nicholas is alive. Assume that* $\delta = 0.065$ *and constant force of mortality* 0.01*. Find the annual benefit premium for this policy.*

Solution:

$$
\begin{aligned}
{}_{25|}\overline{a}_{40} &= \frac{e^{-(25)(0.075)}}{0.075} = 2.044732891, \\
\overline{a}_{40:\overline{25|}} &= \frac{1 - e^{-(25)(0.075)}}{0.075} = 11.28860044, \\
(60000)\overline{P}({}_{25|}\overline{a}_{40}) &= \frac{(60000)(2.044732891)}{11.28860044} = 10867.95251.
\end{aligned}
$$

