Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.
Section 6.8. Non-level premiums and/or benefits.

Extract from:
"Arcones' Manual for SOA Exam MLC. Spring 2009 Edition",
available at http://www.actexmadriver.com/

Non-level premiums and/or benefits.

Let b_k be the benefit paid by an insurance company at the end of year k, $k = 1, 2, \dots$. The contingent cashflow of benefits is

benefits	0	b_1	b_2	b_3	$\cdots$
Time after issue	0	1	2	3	$\cdots$

Hence, the APV of the contingent benefit is

$$\sum_{k=1}^{\infty} b_k v^k \mathbb{P}\{K_x = k\} = \sum_{k=1}^{\infty} b_k v^k \cdot {}_{k-1|}q_x.$$

Let π_{k-1} be the benefit premium received by an insurance company at the beginning of year k, $k = 1, 2, \dots$. The contingent cashflow of benefit premiums is

benefit premiums	π_0	π_1	π_2	π_3	$\cdots$
Time after issue	0	1	2	3	$\cdots$

Hence, the APV of the contingent benefit premiums is

$$\sum_{k=0}^{\infty} \pi_{k+1} v^k \mathbb{P}\{K_x > k\} = \sum_{k=0}^{\infty} \pi_k v^k \cdot {}_k p_x.$$

Under the equivalence principle,

$$\sum_{k=1}^{\infty} b_k v^k \cdot {}_{k-1|}q_x = \sum_{k=0}^{\infty} \pi_k v^k \cdot {}_k p_x.$$

Example 1

For a special fully discrete 15-payment whole life insurance on (20):
(i) The death benefit is 1000 for the first 10 years and is 5000 thereafter.
(ii) The benefit premium paid during the each of the first 5 years is half of the benefit premium paid during the subsequent years.
(iii) Mortality follows the life table for the USA population in 2004.
(iv) $i = 0.06$.
Calculate the initial annual benefit premium.

Solution: Let π be the initial premium. Equating benefits and premiums,

$$2\pi\ddot{a}_{20:\overline{15}|} - \pi\ddot{a}_{20:\overline{5}|} = (1000)A_{20} + (5000)\cdot {}_{10}E_{20}A_{30}.$$

The APV of benefits is

$$\begin{aligned}&(1000)A_{20} + (5000)\cdot {}_{5}E_{20}\cdot {}_{5}E_{25}\cdot A_{30}\\ =&52.456 + (5)(0.743753)(0.743683)(82.295) = 280.0495963.\end{aligned}$$

As to the APV of premiums,

$$\begin{aligned}&\ddot{a}_{20:\overline{15}|} = \ddot{a}_{20} - {}_{15}E_{20}\cdot\ddot{a}_{35} = 16.740 - (1.06)^{-15}\frac{97250}{98709}(15.817)\\ =&10.2376702,\\ &\ddot{a}_{20:\overline{5}|} = \ddot{a}_{20} - {}_{5}E_{20}\cdot\ddot{a}_{25} = 16.740 - (0.743753)(16.514) = 4.457662958\\ &2\pi\ddot{a}_{20:\overline{15}|} - \pi\ddot{a}_{20:\overline{5}|} = ((2)(10.2376702) - 4.457662958)\pi\\ =&16.01767744\pi.\end{aligned}$$

Hence, $\pi = \frac{280.0495963}{16.01767744} = 17.48378299.$

Example 2

Consider a whole life insurance policy to (40) *with face value of 250000 payable at the end of the year of death. This policy will be paid by benefit annual premiums paid at the beginning of each year while* (40) *is alive. Suppose that the premiums increase by 6% each year. Assume that* $i = 6\%$ *and death is modeled using the De Moivre model with terminal age 100. Find the amount of the first benefit annual premium for this policy.*

Solution: We have that

$$A_{40} = \frac{a_{\overline{60}|0.06}}{60} = 0.2693571284$$

and the net single premium is $(250000)(0.2693571284) = 67339.28211$. Let π be the amount of the first benefit premium. Then, $\pi_k = \pi(1.06)^k$, $k = 0, 1, 2, \ldots$ The APV of the benefit annual premiums is

$$\sum_{k=0}^{59} v^k \pi (1.06)^k {}_kp_{40} = \sum_{k=0}^{59} (1.06)^{-k} \pi (1.06)^k \frac{60-k}{60}$$

$$= \sum_{k=1}^{60} \pi \frac{k}{60} = \pi \frac{(60)(61)}{(2)(60)} = 31.5\pi.$$

Hence, $\pi = \frac{67339.28211}{31.5} = 2137.754988$.

Often, insurance products guarantee the return of net annual premiums. Consider a fully discrete unit whole life insurance to (x) that returns the benefit premiums without interest. Let π the annual benefit premium for this insurance. If (x) dies in the k–th year, the death benefit is $1+k\pi$. Hence, the APV of benefits is $A_x+\pi(IA)_x$. To find π, we solve $\pi\ddot{a}_x=A_x+\pi(IA)_x$.

Example 3

Consider a whole life insurance policy to (40) *with pays 250000 plus the return of the annual premiums without interest at the end of the year of death. This policy will be paid by benefit annual premiums paid at the beginning of each year while* (40) *is alive. Suppose that the premiums increase by 6% each year. Assume that* $i = 6\%$ *and death is modeled using De Moivre's model with terminal age 100. Find the amount of the benefit annual premium for this policy using the equivalence principle.*

Solution: Let π be the amount of the benefit annual premium. Using the equivalence principle, $\pi \ddot{a}_{40} = 250000 A_{40} + \pi (IA)_{40}$ and $\pi = \frac{250000 A_{40}}{\ddot{a}_{40} - (IA)_{40}}$. We have that

$$A_{40} = \frac{a_{\overline{60}|0.06}}{60} = 0.2693571284,$$

$$\ddot{a}_{40} = \frac{1 - A_{40}}{d} = \frac{1 - 0.2693571284}{(0.06)/1.06} = 12.90802406,$$

$$(IA)_{40} = \frac{(Ia)_{\overline{60}|0.06}}{60} = 4.253403641,$$

$$\pi = \frac{250000 A_{40}}{\ddot{a}_{40} - (IA)_{40}} = \frac{(250000)(0.2693571284)}{12.90802406 - 4.253403641} = 7780.732007.$$

Consider a fully discrete unit whole life insurance to (x) that returns the benefit premiums with interest if death happens within n years. Let π the annual benefit premium for this insurance. If $K_x \leq n$, then the insurer returns $\pi \ddot{s}_{\overline{K_x}|i}$ at time K_x. The present value at time zero of this payment is $\pi \ddot{a}_{\overline{K_x}|i}$. We have that $\ddot{a}_{\overline{K_x}|i} I(K_x \leq n) + \ddot{a}_{\overline{n}|i} I(K_x > n) = \ddot{a}_{\overline{\min(K_x,n)}|i}$. So,

$$E[\ddot{a}_{\overline{K_x}|i} I(K_x \leq n)] = \ddot{a}_{x:\overline{n}|} - \ddot{a}_{\overline{n}|i} \cdot {}_n p_x.$$

The APV of the return of the benefit premiums is $\pi(\ddot{a}_{x:\overline{n}|} - \ddot{a}_{\overline{n}|i} \cdot {}_n p_x)$. To find π, we solve

$$\pi \ddot{a}_x = A_x + \pi(\ddot{a}_{x:\overline{n}|} - \ddot{a}_{\overline{n}|} \cdot {}_n p_x).$$

Example 4

Consider a special whole life insurance policy to (40) *with pays 250000 at the end of the year of death as well as the return of the annual premiums with interest payable at the end of the year if death happens in the first 15 years. This policy will be paid by benefit annual premiums paid at the beginning of each year while* (40) *is alive. Suppose that the premiums increase by 6% each year. Assume that* $i = 6\%$ *and death is modeled using De Moivre's model with terminal age 100. Find the amount of the benefit annual premium for this policy using the equivalence principle.*

Solution: Let π be the amount of the benefit annual premium. Using the equivalence principle,
$\pi\ddot{a}_{40} = (250000)A_{40} + \pi(\ddot{a}_{40:\overline{15}|} - \ddot{a}_{\overline{15}|} \cdot {}_{15}p_{40})$. and
$\pi = \frac{250000A_{40}}{\ddot{a}_{40}-\ddot{a}_{40:\overline{15}|}+\ddot{a}_{\overline{15}|}\cdot{}_{15}p_{40}}$. We have that

$$A_{40} = \frac{a_{\overline{60}|0.06}}{60} = 0.2693571284,$$

$$\ddot{a}_{40} = \frac{1-A_{40}}{d} = \frac{1-0.2693571284}{(0.06)/1.06} = 12.90802406,$$

$$A_{40:\overline{15}|} = \frac{a_{\overline{15}|0.06}}{60} = 0.1618708165,$$

$$\ddot{a}_{40:\overline{15}|} = \frac{1-0.1618708165}{0.06/1.06} = 14.80694891,$$

$$\ddot{a}_{\overline{15}|} \cdot {}_{15}p_{40} = (10.29498393)\frac{60-15}{60} = 7.721237945,$$

$$\pi = \frac{(250000)(0.2693571284)}{12.90802406 - 14.80694891 + 7.721237945} = 11565.72671.$$

Consider an n–year deferred life annuity which pays B at the end of the year of death plus a return with interest of the payments made if death happens in the deferral period. Let π be the amount of the benefit annual premium. Using the equivalence principle,

$$\pi \ddot{a}_{x:\overline{n}|} = B \cdot {}_{n}|\ddot{a}_x + \pi(\ddot{a}_{x:\overline{n}|} - \ddot{a}_{\overline{n}|} \cdot {}_{n}p_x).$$

Hence, $\pi \ddot{a}_{\overline{n}|} \cdot {}_{n}p_x = B \cdot {}_{n}|\ddot{a}_x$ and

$$\pi = \frac{B \cdot {}_{n}|\ddot{a}_x}{\ddot{a}_{\overline{n}|} \cdot {}_{n}p_x} = \frac{B \cdot {}_{n}E_x \ddot{a}_{n+x}}{\ddot{a}_{\overline{n}|} \cdot {}_{n}p_x} = \frac{B \cdot {}_{n}p_x v^n \ddot{a}_{n+x}}{\ddot{a}_{\overline{n}|} \cdot {}_{n}p_x} = \frac{B v^n \ddot{a}_{n+x}}{\ddot{a}_{\overline{n}|}} = \frac{B \ddot{a}_{x+n}}{\ddot{s}_{\overline{n}|}}.$$

Example 5

A 20–year deferred life annuity on (40) is funded by annual benefit premiums at the beginning of the first 20 years while (40) is alive. This insurance pays 250000 at the end of the year of death plus the return of the annual premiums with interest if death happens during the deferral period. Assume that $i = 6\%$ and death is modeled using De Moivre's model with terminal age 100. Find the amount of the benefit annual premium for this policy using the equivalence principle.

Solution: We have that

$$A_{60} = \frac{a_{\overline{40}|0.06}}{40} = 0.3761574218,$$

$$\ddot{a}_{60} = \frac{1 - A_{60}}{d} = \frac{1 - 0.3761574218}{(0.06)/1.06} = 11.02121888,$$

$$\ddot{s}_{\overline{20}|} = 38.99272668,$$

$$\pi = \frac{B\ddot{a}_{60}}{\ddot{s}_{\overline{20}|}} = \frac{(250000)(11.02121888)}{38.99272668} = 70662.01711.$$

