Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.

Section 6.9. Computing benefits premiums from a life table.

Extract from:
"Arcones' Manual for SOA Exam MLC. Spring 2009 Edition",
available at http://www.actexmadriver.com/

Fully discrete insurance.

We assume that the death benefits are paid at the end of the year of death. But, the benefit premiums are at the beginning of each m–thly period. The annual benefit premium in this situation is higher than the regular situation. Benefit premiums, instead of being received at the beginning of the year, they are received later on. During a year when an insuree dies, benefit premiums may not be received during the whole year. From a life table, we can find $\ddot{a}_x$, then we can estimate $\ddot{a}_x^{(m)}$.

Theorem 1

Assuming a uniform distribution of deaths, we have that:

(i) $\overline{A}_x = \frac{i}{\delta} A_x$.

(ii) $\overline{A}^{1}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|}$.

(iii) ${}_{n|}\overline{A}_x = \frac{i}{\delta} \cdot {}_{n|}A_x$.

(iv) $\overline{A}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \ 1}$.

Theorem 2

Assuming a uniform distribution of deaths, we have that:

(i) $A^{(m)}_x = \frac{i}{i^{(m)}} A_x$.

(ii) $\overset{(m)}{A}{}^{1}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|}$.

(iii) ${}_{n|}A^{(m)}_x = \frac{i}{i^{(m)}} \cdot {}_{n|}A_x$.

(iv) $A^{(m)}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \ 1}$.

Whole life annuities

Recall that:

$$\ddot{a}_x = \frac{1 - A_x}{d},$$

$$a_x = \frac{v - A_x}{d},$$

$$\bar{a}_x = \frac{1 - \bar{A}_x}{\delta},$$

$$\ddot{a}_x^{(m)} = \frac{1 - A_x^{(m)}}{d^{(m)}},$$

$$a_x^{(m)} = \frac{v^{1/m} - A_x^{(m)}}{d^{(m)}}.$$

Temporary annuities

Recall that

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d},$$

$$\overline{a}_{x:\overline{n}|} = \frac{1 - \overline{A}_{x:\overline{n}|}}{\delta},$$

$$\ddot{a}^{(m)}_{x:\overline{n}|} = \frac{1 - A^{(m)}_{x:\overline{n}|}}{d^{(m)}},$$

$$a^{(m)}_{x:\overline{n}|} = \ddot{a}^{(m)}_{x:\overline{n}|} - \frac{1}{m} + \frac{1}{m}{}_nE_x,$$

Benefit premiums paid m times a year

For a whole life insurance to (x), the annual benefit premium is

$$P_x^{(m)} = \frac{A_x}{\ddot{a}_x^{(m)}}.$$

For an n–year endowment to (x), the annual benefit premium is

$$P_{x:\overline{n}|}^{(m)} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(m)}}.$$

For a whole life insurance to (x) funded for h years, the annual benefit premium is

$${}_hP_x^{(m)} = \frac{A_x}{\ddot{a}_{x:\overline{h}|}^{(m)}}.$$

For an n–year endowment to (x) funded for h years, the annual benefit premium is ${}_hP_{x:\overline{n}|}^{(m)} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{h}|}^{(m)}}$.

Theorem 3

Under a uniform distribution of death on each year of death:

$$P_x^{(m)} = \frac{P_x}{\alpha(m) - \beta(m)(P_x + d)}$$

$$P_{x:\overline{n}|}^{(m)} = \frac{P_{x:\overline{n}|}}{\alpha(m) - \beta(m)(P_{x:\overline{n}|}^{1} + d)}$$

$${}_nP_x^{(m)} = \frac{{}_nP_x}{\alpha(m) - \beta(m)(P_{x:\overline{n}|}^{1} + d)}$$

$${}_hP_{x:\overline{n}|}^{(m)} = \frac{{}_hP_{x:\overline{n}|}}{\alpha(m) - \beta(m)(P_{x:\overline{n}|}^{1} + d)}.$$

where $\alpha(m) = \frac{id}{i^{(m)}d^{(m)}}$ *and* $\beta(m) = \frac{i-i^{(m)}}{i^{(m)}d^{(m)}}$.

Proof: Using that $\ddot{a}_x^{(m)} = \alpha(m)\ddot{a}_x - \beta(m)$ and $P_x = \frac{1-d\ddot{a}_x}{\ddot{a}_x}$,

$$P_x^{(m)} = \frac{A_x}{\ddot{a}_x^{(m)}} = \frac{A_x}{\alpha(m)\ddot{a}_x - \beta(m)} = \frac{P_x}{\alpha(m) - \beta(m)(P_x + d)}.$$

We have that

$$P^1_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1 - d\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \text{ and } P^1_{x:\overline{n}|} + d = \frac{1}{\ddot{a}_{x:\overline{n}|}}.$$

Hence,

$$P^{(m)}_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}} = \frac{A_{x:\overline{n}|}}{\alpha(m)\ddot{a}_{x:\overline{n}|} - \beta(m)} = \frac{P_{x:\overline{n}|}}{\alpha(m) - \beta(m)(P^1_{x:\overline{n}|} + d)}.$$

The rest of the expressions can be proven similarly.

Example 1

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0
d_x	33	56	54	45	34	28	0

Assume that $i = 6.5\%$ *and uniform distribution of deaths over each year of death.*
(i) Find $P_{80}^{(12)}$, *using that* $P_{80} = 0.2710105645$.
(ii) Find $P_{80}^{(12)}$, *using that* $A_{80} = 0.8161901166$.

Solution: (i) We have that

$$d = 0.065/1.065 = 0.06103286385, i^{(12)} = 0.06314033132,$$

$$d^{(12)} = 0.06280984512,$$

$$\alpha(12) = \frac{id}{i^{(12)}d^{(12)}} = \frac{(0.065)(0.06103286385)}{(0.06314033132)(0.06280984512)} = 1.000328233$$

$$\beta(12) = \frac{i - i^{(12)}}{i^{(12)}d^{(12)}} = \frac{0.065 - 0.06314033132}{(0.06314033132)(0.06280984512)} = 0.468922420$$

$$P_{80}^{(12)} = \frac{P_{80}}{\alpha(12) - \beta(12)(P_{80} + d)}$$

$$= \frac{(0.2710105645)}{1.000328233 - (0.4689224203)(0.06103286385 + 0.2710105645)}$$

$$= 0.3208647198.$$

Solution: (ii) We have that

$$A_{80}^{(12)} = \frac{i}{i^{(12)}} A_{80} = \frac{0.065}{0.06314033132}(0.8161901166) = 0.8402293189,$$

$$\ddot{a}_{80}^{(12)} = \frac{1 - A_{80}^{(12)}}{d^{(12)}} = \frac{1 - 0.8402293189}{0.06103286385} = 2.543720348,$$

$$P_{80}^{(12)} = \frac{0.8161901166}{2.543720348} = 0.3208647198.$$

Semicontinuous insurance.

For a semicontinuous insurance, the death benefit is paid at the time of the death. The present value of the death benefit is $\overline{A}_x$. From a life table, we can find A_x and P_x. Then, we can estimate $\overline{A}_x$ and $P(\overline{A}_x)$.

Theorem 4

Under a uniform distribution of death on each year of death:

$$P(\overline{A}_x) = \frac{i}{\delta} P_x$$

$$P(\overline{A}^{1}_{x:\overline{n}|}) = \frac{i}{\delta} P^{1}_{x:\overline{n}|},$$

$$P({}_{n}|\overline{A}_x) = \frac{i}{\delta} \cdot {}_{n}|P_x,$$

$$P(\overline{A}_{x:\overline{n}|}) = \frac{i}{\delta} P^{1}_{x:\overline{n}|} + P_{x:\overset{1}{\overline{n}|}},$$

Proof: Using that $\overline{A}_x = \frac{i}{\delta} A_x$, $P(\overline{A}_x) = \frac{\overline{A}_x}{\ddot{a}_x} = \frac{\frac{i}{\delta} A_x}{\ddot{a}_x} = \frac{i}{\delta} P_x$. The rest of the formulas can be proven similarly.

Example 2

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0
d_x	33	56	54	45	34	28	0

Assume that $i = 6.5\%$ and uniform distribution of deaths over each year of death. Find $P(\overline{A}_{80})$ using that $P_{80} = 0.2710105645$.

Solution: We have that

$$P(\overline{A}_{80}) = \frac{i}{\delta} P_{80} = \frac{0.065}{\ln(1.065)}(0.2710105645) = 0.2797259686.$$

Fully continuous insurance.

For a fully continuous insurance, we need to know $\overline{A}_x$ and $\overline{a}_x$. From a life table, we can find A_x and a_x. Then, we need to estimate $\overline{A}_x$ and $\overline{a}_x$.

Theorem 5

Under a uniform distribution of death on each year of death:

$$\overline{P}(\overline{A}_x) = \frac{\frac{i}{\delta} A_x}{\alpha(\infty)\ddot{a}_x - \beta(\infty)} = \frac{\frac{i}{\delta} P_x}{\alpha(\infty) - \beta(\infty)(d + P_x)},$$

$$\overline{P}(\overline{A}^1_{x:\overline{n}|}) = \frac{\frac{i}{\delta} A^1_{x:\overline{n}|}}{\alpha(\infty)\ddot{a}_{x:\overline{n}|} - \beta(\infty)(1 - {}_nE_x)},$$

$$\overline{P}(\overline{A}_{x:\overline{n}|}) = \frac{\frac{i}{\delta} A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}}{\alpha(\infty)\ddot{a}_{x:\overline{n}|} - \beta(\infty)(1 - {}_nE_x)},$$

where $\alpha(\infty) = \frac{id}{\delta^2}$ *and* $\beta(\infty) = \frac{i-\delta}{\delta^2}$.

Proof: Using that $\overline{A}_x = \frac{i}{\delta} A_x$, $\overline{a}_x = \alpha(\infty)\ddot{a}_x - \beta(\infty)$ and $P_x = \frac{A_x}{\ddot{a}_x} = \frac{1-d\ddot{a}_x}{\ddot{a}_x}$,

$$\overline{P}(\overline{A}_x) = \frac{\overline{A}_x}{\overline{a}_x} = \frac{\frac{i}{\delta} A_x}{\alpha(\infty)\ddot{a}_x - \beta(\infty)} = \frac{\frac{i}{\delta} P_x}{\alpha(\infty) - \beta(\infty)\frac{1}{\ddot{a}_x}}$$

$$= \frac{\frac{i}{\delta} P_x}{\alpha(\infty) - \beta(\infty)(d + P_x)}.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0
d_x	33	56	54	45	34	28	0

Assume that $i = 6.5\%$ *and uniform distribution of deaths over each year of death.*
(i) Find $\overline{P}(\overline{A}_{80})$, *using that* $P_{80} = 0.2710105645$.
(ii) Find $\overline{P}(\overline{A}_{80})$.

Solution: (i) We have that

$$\alpha(\infty) = \frac{id}{\delta^2} = \frac{(0.065)(0.065/1.065)}{(\ln(1.065))^2} = 1.000330529,$$

$$\beta(\infty) = \frac{i-\delta}{\delta^2} = \frac{0.065 - \ln(1.065)}{(\ln(1.065))^2} = 0.5106631458,$$

$$\frac{i}{\delta} = \frac{0.065}{\ln(1.065)} = 1.032158909,$$

$$\overline{P}(\overline{A}_{80}) = \frac{\frac{i}{\delta}P_{80}}{\alpha(\infty) - \beta(\infty)(d + P_{80})}$$

$$= \frac{(1.032158909)(0.2710105645)}{1.000330529 - (0.5106631458)((0.065/1.065) + 0.2710105645)}$$

$$= 0.3367076073.$$

Solution: (ii) We have that

$$\overline{A}_{80} = \frac{i}{\delta} A_{80} = \frac{0.065}{\ln(1.065)}(0.8161901166) = 0.8424379003,$$

$$\overline{P}(\overline{A}_{80}) = \frac{0.8424379003}{\frac{1-0.8424379003}{\ln(1.065)}} = 0.3367076072.$$

