Study Guide for Schaum's Outlines
Calculus
fourth edition

for
Calculus I

December 9, 2007

Chapter 0

Introduction

The chapters covered in this course are 1–24, 29 and 30 for a total of 26 chapters.

All chapters in the text are divided into three parts:

1. A section of narrative text containing from 2 through 7 pages.
2. A section of solved problems numbered from 1 and containing from 3 to 27 problems complete with solutions and explanations.
3. A section of supplementary problems for the student to work on. These are numbered consecutively after the last solved problem. Final answers to many of the supplementary problems are given, but full solutions are not supplied.

Some of the material to be learned will be covered in the narrative part (1 above) and much will be covered in the solved problems (2 above). The student is expected to work through the solved problems before attempting to work on the supplementary problems. The fact that the sovled problems contain part of the material to be learned accounts for the fact that the narrative section is often quite short.

Chapter 1

Linear coordinate systems. Absolute value. Inequalities

You are expected to learn

1. Types of intervals on the real line (Page 3) and interval notation (Page 3 and Solved Problem 1).
2. The use of the absolute value and inequalities in expressing intervals on the real line (Solved Problem 2).
3. Solution of inequalities of various levels of complexity (see Solved Problems 4 and 9 for factoring problems, Solved Problems 5 and 8 for absolute values needing no break into cases, and Solved Problems 6 and 7 for absolute values needing breaks into cases).

Chapter 2

Rectangular coordinate systems

This will vary a lot with the instructor. Through the top of Page 13 and Solved Problems 1–3 is a minimum.

Chapter 3

Lines

Every student comes in knowing the slope-intercept equation of a line (bottom of Page 21). Of more importance in this course is the point-slope equation of a line (middle of Page 21) and the student that does not learn how to use this easily will be in trouble.

All material to the top of page 23 should be learned.

Solved Problem 1 is basic, Solved Problems 2 and 3 are good and Solved Problem 4 is better for the end material in Chapter 2 than the Solved Problems in Chapter 2.

Chapter 4

Circles

All material to the top of Page 32 is crucial especially the use of "completing the square" in Example 2. Completing the square shows up in at least four different places in this course and needs to be learned cold. Problems with circles can be difficult and all Solved Problems should be studied.

Chapter 5

Equations and their graphs

The definition that starts the chapter should be memorized and understood. The rest of the material is about graphs of quadratic equations (the conic sections discussed on Page 42).

Solved Problems 1, 2, 3 and 18 are too easy to be useful, but you should know what the graphs look like. Unfortunately, these problems give the student the impression that "plotting a few values" is a useful technique in this course. It is not.

Solved Problems 4, 7 and 10 are more useful as background knowledge. Solved Problems 5 and 6 discuss concepts that are no longer emphasized in calculus courses and can be skipped.

Solved Problems 8, 9, 11–14, 16 and 17 are the kind of work problems you must be able to solve. This chapter is the second place where completing the square is important and and Solved Problem 17 shows how it comes in.

Solved Problem 15 is so important that I am putting it in a separate paragraph.

Chapter 6

Functions

All text material to the bottom of page 54 and all Solved Problems are useful.

Chapter 7

Limits

The notation in the definition of limit in the second paragraph of the chapter can be compared to the terminology introduced in Solved Problems 2d and 2f on Page 5.

The important first part is to be able to get the right values for limits. [The first two sentences in the chapter, the two sentences before Fig. 7-1, Example 1, Example 3, Theorems 7.1–7.6, the discussion headed by the word "Infinity" starting at the bottom of Page 62, Examples 4, 5, the discussion after Example 5, Example 6, Solved Problems 1–8.]

The second part is to use the theory behind the calculations. This can be limited to arguments such as given in Example 2 and Solved Problem 9.

Students planning to major in math should try for a better understanding of the concepts by looking at the remaining Solved Problems.

Chapter 8

Continuity

The definition (first four lines of the chapter) should be memorized (and compared to the two sentences before Figure 7-1 on Page 61). The notions of *removable* (top of Page 72) and *jump* (top of Page 73) discontinuties should be understood.

Theorems 8.1–8.4 say that there are a lot of continuous functions.

All examples are relevant.

The material starting with the definition on Page 74 gives important consequences of continuity that will be used repeatedly.

Solved Problem 1 reviews the material through Example 4.

Chapter 9

The derivative

This chapter teaches a set of calculations known as "calculating the derivative from the definition." [In the next chapter you will learn "calculating the derivative from the rules."]

The very short narrative on Pages 79 and 80 should be learned, but the bulk of the technique to be learned is in Solved Problems 3–8. See also Solved Problem 11.

The discussions in Solved Problems 9 and 10 are important.

Chapter 10

Rules for differentiating functions

This chapter covers the "other" way to calculate derivatives. This is the technique that you will use almost all of the time for computing derivatives in this course.

There are 9 rules in Theorem 10.1, the chain rule discussed on Page 87, and the rule for differentiating inverse functions given in Theorem 10.2(b). [Note that 10.2(a) uses the words *increasing* and *decreasing* that seem not to have been defined before this point, so 10.2(a) can be safely ignored.]

Higher derivatives are introduced on Page 89 and usually give students no problems.

The chain rule is fantastically important and it is easy to miss where it needs to be used.

Solved Problems 5–8 do not use the chain rule, Solved Problems 9, 11 and 14 make light use of the chain rule, and Solved Problems 12, 16, 20 and 23 make heavier use of the chain rule. Some Solved Problems after Solved Problem 10 make use of Solved Problem 10. In Solved Problems 20 and 23, the use is obvious. Solved Problem 15 illustrates the quotient rule. Other Solved Problems to look at are 18, 21, 22.

Solved Problem 13 had bad luck on the way to the printer.

All the techniques in this chapter are supposed to be automatic to the student in a very short time and the goal is to come to regard all the problems from this chapter as extremely easy.

Chapter 11

Implicit differentiation

The subject of this chapter is a very standard technique. All the narrative and all Solved Problems are relevant.

Chapter 12

Tangent and normal lines

This is very standard. It is also where the point-slope formula for the equation of a straight line becomes important. The formula in Item 3 in the middle of Page 103 need not be memorized.

Solved Problems 1 and 2 are basic, Solved Problems 3–6 require a bit more work and Solved Problems 7 and 8 are a bit more of a workout.

Chapter 13

Law of the mean. Increasing and decresasing functions

The Law of the Mean is usually called the Mean Value Theorem (MVT) in more recent texts.

The point of learning derivatives is that they are useful, and much of the usefulness of the derivative comes from the MVT. This is true in spite of the fact that the statement of the MVT does not look very useful.

This chapter emphasizes argument. The easiest way to get used to arguments is to memorize certain examples. These are found in Solved Problems 5–7 and 9.

Learn the statements of Theorems 13.1, 13.2, 13.4. Corollary 13.3 is not important enough to worry about and we will ignore 13.5 and 13.6.

The proof of 13.1 is in Solved Problem 5, the proof of 13.2 is in Solved Problem 6 and the proof of 13.4 is in Solved Problem 7. These should be learned.

The first consequence of the MVT is Theorem 13.7 about increasing and decreasing functions. [Note that this part supplies the definitions missing from Theorem 10.2(a).] The proof is in Solved Problem 9 which should be learned.

There are also several important definitions in this chapter that must be learned: *releative maximum*, *relative minimum* and *relative extremum* on Page 108, *increasing* and *decreasing* on Page 110.

There was supposed to be a Theorem 13.8 because one is referred to in Solved Problem 9 of Chapter 14. It leaves a gap in the logic, but we will do without it.

Practical problems are worked out in Solved Problems 10–12.

Chapter 14

Maximum and minimum values

This is a large chapter and all of it has to be learned.

There are several definitions to learn: *critical number* on Page 115, *absolute maximum* and *absolute minimum* and *absolute maximum* on Page 117. The term *inflection point* occurs on Page 117, but its definition is not given until the next chapter.

The content of the chapter is a collection of techniques for finding relative extrema and absolute extrema under a variety of circumstances.

Relative extrema are easier and have two techniques to choose from for their detection: the second derivative test (Page 115) and the first derivative test (Page 116). The second derivative test often involves less work but does not always give information. You must learn the conditions under which the second derivative test gives useful information and the conditions under which it does not. Examples 2 and 3 are relevant, but you can ignore the comment about inflection point at the very end of (c) in Example 3. Solved Problems 2–7 are relevant.

Absolute extrema form a larger topic. There are some situtations where the solution is very straightforward and then there are the "remaining cases" where the solution is more involved.

Absolute extrema on a closed interval is one nice situation and is covered from the middle of Page 117 to the top third of Page 118, including Example 4.

Absolute extrema when there is only one relative extremum is another nice case and is discussed in the middle of Page 118.

The two situations above cover most of the Solved Problems discussed in the chapter that ask for absolute maxima, but they don't cover all possible situations. This will become more clear in the next chpater.

Searches for absolute maxima are often tied into word problems. Solved Problems 10-22 illustrate this. Two important steps in the solution of any such problem are (1) turning the words of the problem into a function whose extrema are desired, and (2) writing down the domain of the function that is relevant to the wording of the problem. (For example, a negative area might make no sense in an area problem.) The task of finding the extrema using the techniques of the chapter cannot begin until steps (1) and (2) are done. In the Solved Problems listed, step (1) is done without much explanation and (2) is often not mentioned, but should be. You should fill in anything missing when reading the Solved Problems.

Chapter 15

Curve sketching. Concavity. Symmetry

This is another large chapter. It teaches how to determine several aspects of a graph. Each aspect comes with its own technique: concavity is discussed on Page 129 and in Example 1 on Page 130; points of inflection (mentioned without definition in Example 3(c) in Chapter 14) are discussed right after through Example 2; vertical and horizontal asymptotes are discussed through Example 3; symmetry of several forms through Example 6 which is related to the topic of even and odd functions which finish out the narrative on Page 132. We will ignore oblique asymptotes which are discussed in Solved Problem 7.

The main difficulty in this topic is that a typical problem asks the student to gather all the information from the techniques listed above and use the information to draw a graph that agrees with the information gathered. The summary of how to proceed on Page 133 addresses this. (Step 11 which is about oblique asymptotes can be ignored.) It is harder than it looks and requires practice. The length of the list hints at the difficulty. All Solved Problems are relevant, but the oblique asymptote aspects of Solved Problems 7 and 10 can be omitted.

Chapter 16

Review of trigonometry

This is supposed to be review, but will be taught as if you have forgotten most of it. You are expected to know what the six trig functions (sin, cos, tan, cot, sec, csc) are in terms of the sides of a right triangle, and hopefully you remember that $\sin^2(x) + \cos^2(x) = 1$.

In a very short time you should know the table in Fig. 16-2 and Table 16-1 (why one is a "Fig." and one is a "Table" is hard to fathom) backwards and forwards.

You should also learn all of the identities in (16.1) through (16.14) although it has to be admitted that many will not see use until Calculus II. The law of cosines and the law of sines (16.15) are beautiful, but hardly ever come up in a calculus course.

Solved Problems 1–7 are relevant. Solved Problems 8–10 discuss polar coordinates which will can be ignored for now since the topic will be covered completely in Calculus II.

This chapter only discusses sin and cos. The next chapter is partly a continuation of this chapter and will discuss the other four trig functions.

Chapter 17

Differentiaion of trigonometric functions

The differentiation material in this chapter breaks into three parts. Knowing the derivatives of the six trig functions, knowing how to prove that the derivatives of the six trig functions are correct, and using the derivatives of the six trig fuctions to calculate the derivatives of complicated expressions involving the trig functions.

The rest of the chapter is a continuation of the previous chapter and it covers tan, cot, sec, and csc as well as their derivatives.

Knowing the derivatives is an absolute must. They are listed in (17.3)–(17.8).

The derivation of the derivative of the sin is involved and should be learned assuming (17.1). The proof uses (17.2) (which is proven on Page 153) and the rest of the proof is in Solved Problem 2. Prospective math majors can look at the proof of (17.1) in Solved Problem 1. Everyone else just has to know that (17.1) is true. The derivative of cos is computed on Page 153 just after (17.4).

The formulas for tan, cot, sec, and csc are given near the bottom of Page 155 just above (17.5). The derivatives of these four functions can be computed from these formulas using the quotient rule and the derivatives of sin and cos. The derivatives themselves are given in (17.5)–(17.8).

The rest of the chapter gives information about the graphs of the six trig functions and some identities (17.9)–(17.11) about tan, cot, sec, and csc.

The material on page 158 about the angle of inclination and angles between curves can be skipped.

Solved Problems 2 and 3 are relevant, but the derivation of the derivative of sec on 3(b) is hardly any easier than a direct calculation along the lines of 3(a). Solved Problems 6, 9, 10, 11, 12 give exercise with derivatives.

Solved Problem 8 is a standard technique for finding certain limits assuming that (17.1) is known. The technique should be learned.

Solved Problem 13 really belongs in a later chapter.

Solved Problems 6, 14, 15 and 16 use techniques from previous chapters coupled with information about the trig functions.

Chapter 18

Inverse trigonometric functions

This chapter has two parts: (1) what are the inverse trig functions, and (2) what are their derivatives.

The key to (1) is to be familiar with the graphs of the six trig functions and to know what is expected of the inverse function. The restrictions on the ranges of the six inverse functions should then be "obvious." You should work to get to the point where the definitions of the six inverse trig functions is obvious. Even if the reason for the definitions never becomes obvious, you should commit all six of them to memory.

Calculating the derivatives of the inverse trig functions is a standard exercise using implicit diff. These standard exercises involve another type of standard exercise: the computation of expressions such as $\sin(\cos^{-1}(x))$ and others like it. However, the calculations in the book avoid pointing this out. The student should try various combinations like this and also try problem 35(a).

The first sentence below the figure on Page 170 is not self explanatory. It might be explained by the instructor or might not. It is not critical.

The second sentence below the figure on Page 170 is very important. It introduces the notation arcsin, arccos, etc., but $\sin^{-1}(x)$ is still used everywhere and must be gotten used to.

The hard part of the chapter is getting used to the fact that $y = \sin^{-1}(x)$ has the same meaning as $x = \sin(y)$ as long as y is in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. The part of the sentence before the words "as long as" and the part of the sentence after the words "as long as" are equally important.

Solved Problem 1 does a caclulation omitted from the narrative part.

Solved Problems 2–9 are exercises in computing derivatives.

Solved Problems 10, 11, 12, 13 emphasize the basic concepts of the inverse trig functions.

Chapter 19

Rectilinear and circular motion

This is a chapter of definitions. Certain words mean certain things connected with differentiation. You learn what the words mean and what to do when they come up in problems. Study everything in the narrative part, memorizing all *italicized terms*, and seeing how they are used in the examples. All Solved Problems are relevant.

Chapter 20

Related rates

The subject of this chapter is a very standard technique. All the narrative and all Solved Problems are relevant.

The key to the chapter is the sentence: "If quantities are related, then their rates of change are related."

All the problems are word problems and the important steps of each of them are: (1) write down an equation that relates the important quantities, (2) record what is given about the rate of change of some of them, (3) identify the rate of change that is unknown, (4) differentiate the equation from (1) with respect to time, (5) plug in the known rates from (3), and then (6) solve for the unknown rate.

Read the Solved Problems to see where the six steps given above come into play in each of them.

Chapter 21

Differentials. Newton's method

This chapter is an amalgam of two topics.

The first is based on display (21.2), which comes from the all important (21.1), and the uses of (21.2) as illustrated in Examples 1 and 2. The notion of a differential is best identified as "an approximation to a change in a function" and the approximation used is the one in (21.1) or the definition in the two lines at the top of Page 189. The discussion on the rest of Page 189 muddies the fact that the approximation used follows the tangent line.

The second topic is Newton's Method which is only related in that the tangent line is used here as well. The key formula is (21.5) together with its very important minus sign. Some find it easier to remember the derivation of x_1 from x_0 that is given in the half dozen lines after Fig. 21-2. Example 3 shows how Newton's Method is used.

Solved Problems 1–3, 5 and 6 are relevant.

Chapter 22

Antiderivatives

This chapter introduces the reverse of the process of differentiation. It starts with simple problems and rules (through Examples 3 and 4), and goes on to more complicated problems.

Example 1 emphasizes that all answers to indefinite integrals must have the "$+C$" to be fully correct.

Law (8) is the reverse of the chain rule applied to powers. It is illustrated in Examples 5 and 6.

The full reverse of the chain rule is Law (9) which is not adequately illustrated in the text. Example 7(a) is a very straightforward illustration and 7(b) is a tiny step toward illustrating its flexibility. The rest of Page 198 is just a list known differentiation facts down to the last 3 which are unjustified and can be checked by differentiating. The technique to derive the last three should be gone over in class. This removes dependence on remembering lots of details.

The important fact that any answer to an indefinite integral can be checked by differentiating seems not to be mentioned in the text.

The guess and adjust technique for simple integrals is not covered in the text and should be covered in class.

The fact that very different looking answers can be correct answers for the same problem is not covered. For example Problem 23 has the answer

$$\frac{x^2}{x+1} + C$$

as the answer. But for (say) $C = 1$, the function of x part of the answer becomes

$$\frac{x^2 + x + 1}{x+1}$$

and for $C = 2$ it becomes

$$\frac{x^2 + 2x + 2}{x+1} = \frac{(x+1)^2 + 1}{x+1}$$

and so forth. Further manipulation shows all of

$$\frac{x^2}{x+1} + C,$$
$$\frac{x^2+x+1}{x+1} + C,$$
$$\frac{1}{x+1} + x + C,$$

and many more are valid answers to Problem 23. In later chapters $\sin^2(x) + C$ and $-\cos^2(x) + C$ can both be valid answers to a single antidifferentiation problem.

Solved Problems 1–4 emphasize the power rule and show that minor algebra might be needed to invoke it. Solved Problem 5 illustrates sums and differences.

Solved Problems 6–8 show that less minor algebra might be needed to set things up so that rules can be used.

Solved Problems 9–15 illustrate Law (8).

Law (9) is illustrated in 16–18 with 16 and 17 being very straightforward and 18 being less so.

Solved Problem 18 is crucial to many of the Supplementary Problems's in this chapter.

Solved Problem 19 illustrates that when differentiation is turned around, the words associated to differentiation can also be worked with in the reverse of the usual order. It also illustrates the use of the "$+C$" (which is never given its proper name *constant of integration* in the chapter).

Solved Problem 20 turns a definite integral into a word problem in a rather trivial way.

Nothing in the text illustrates the use of completing the square. This needs to be covered in class and students need to take careful note of it. Supplementary Problems's such as 39, 40 and others need the technique.

It should be noted that the only thing in this chapter that is not mentioned above is Solved Problem 21 which students can read or ignore depending on their level of interest. All other parts of this chapter are not optional.

Chapter 23

The definite integral. Area under a curve

This a large chapter. All of the text, and Solved Problem's 1–4 are relevant. Solved Problem 5 is also nice to know how to do, but not critical. Supplementary Problem 12 proves the formula needed in Solved Problem 4, but is not critical either.

Part of the point of the chapter is that area problems are hard to do directly and life becomes easier when the indirect method of the next chapter becomes available. This can be compared to doing derivatives from the definition in Chapter 9 as compared to doing derivatives from the rules in Chapter 10.

However, another point of the chapter is that the areas have nice behavior. Two different regions will have a total area that is the sum of the two areas, areas below the x-axis are negative and so forth. This nice behavior is summarized in the "Properties of the definite integral" section starting on Page 210 and continuing into Solved Problem's 1–3.

Chapter 24

The fundamental theorem of calculus

This chapter relates the material in the previous two chapters. The idea is that doing definite integrals from the definition (Chapter 23) is harder than anti-differentiation (Chapter 22). This chapter says that anti-differentiation can be used to do definite integrals. This has two consequences.

One is that lots of definite integrals can be given that could not have been done by the techniques of Chapter 23. This is covered from the middle of Page 217 through Example 1.

The form that this takes when subtitution is used can get involved and the material on Page 218 through Example 2 shows an important way to cut down on the writing.

The second consequence is the rather tiny remark leading to display (24.2) on Page 217. This is expanded without explanation into a series of Supplementary Problems (27–32) which are worth knowing how to do.

There are two topics "Mean-value theorem for integrals" and "Average value of a function on a closed interval" at the beginning of the chapter. The "Average value of a function on a closed interval" is worth learning.

Solved Problem's 1–3 review the basic material.

Solved Problem 6 introduces important short cuts.

Solved Problem 7 is an important approximation technique that could have been in the previous chapter.

Chapter 29

Applications of integration I: Area and arc length

All of the material must be learned.

The examples in the text about area do not show the complexities of problems that are usually given. Solved Problem's 1–6 show the difficulties that can occur. Solved Problem 5 is especially important in that it shows that signs can be a problem.

Random arc length problems are impossible to do. The problems that can be done are carefully rigged to work out. Solved Problem 7 and 8 show how this happens. Solved Problem 9 is to be ignored since it uses material from the chapters that were skipped.

Chapter 30

Applications of integration II: Volume

This chapter requires much practice. Problems can be attacked by several methods. Often a problem can be attacked by two of the methods. Often a problem will be easier to do with one method than another. Sometimes a problem is *much* easier to do with one method than another. And every now and then a problem is possible with one method and not another. The bottom line is that a student has to be willing to try more than one method if the first try runs into trouble.

All problems involve setting up one or more integrals to calculate a volume and then doing the integrals. The important step is the first one: setting up the integral(s). There are a lot of details in setting up the integral(s) correctly and it takes practice to get all the details right.

Most volumes are gotten by rotating an area. Most areas can be thought of as "painted" by a line being dragged in a certain direction. The type of integral (as well as the variable of integration) is determined by the direction of the line and whether it is parallel or perpendicular to the axis of rotation. The student should learn how.

Solved Problem's 1–8 all illustrate the techniques applicable to solids of rotation: disk, washer and shell. Solved Problem's 12 and 13 illustrate slice method which applies to solids not generated by rotation.