Math 222, Practice Final Exam/Review Sheet, Fall 2007

NOTE: This sheet should not be your only study guide for the final! It only represents a minimum of concepts you should have mastered as a result of taking Math 222.

1. (You should know the various techniques of integration — all of Chapter 8.) Calculate the following integrals - give numerical answers for the definite integrals:

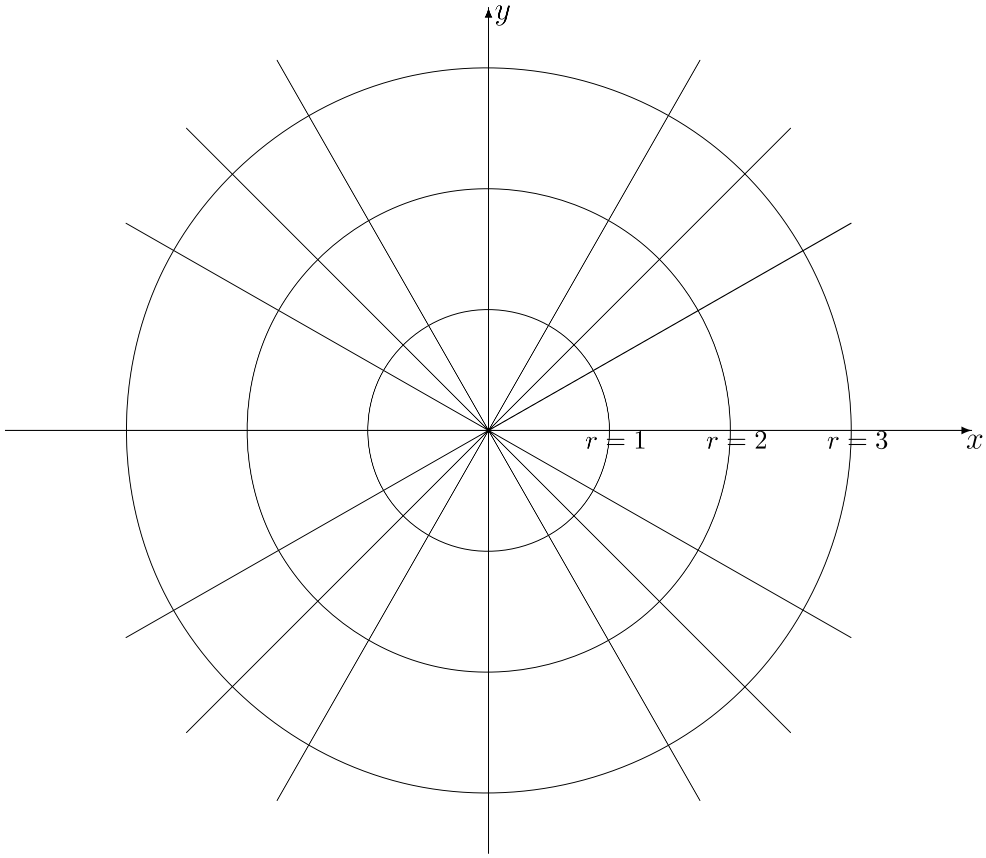
   (i) $\displaystyle\int_0^\infty \frac{x^2}{4+x^6}\,dx$

   (ii) $\displaystyle\int x\sec x\tan x\ dx$

(iii) $\displaystyle\int \frac{1}{x^3 + x}\, dx$

(iv) $\displaystyle\int (x-1)\sqrt{x^2 - 2x - 3}\, dx$

2. (You should know everything about polar coordinates — Section 11.3.) Sketch the curve with the polar equation $r = 1 - 2\sin\theta$ on the polar graph paper below.



3. (You should know infinite series, especially all the tests for convergence/divergence — Sections 12.2–12.7.) Determine whether the series converges or diverges. Make sure to show your work and state what tests you use to prove your answer.

(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^{1/n}}{n}$

(ii) $$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n}$$

(iii) $$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^3\,10^n}{n!}$$

4. (You should know power series, Taylor series (at any center point a), Maclaurin series — Sections 12.8–12.10.) Power series problems.

   (i) Find the interval of convergence and the radius of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3\,(2x+1)^n}{5^n}$.
   Make sure to show your work and state what tests you use to find your answers.

(ii) Find a power series representation for $\int x^2\sqrt{1+x^2}dx$.

(iii) Find the Maclaurin series for $x^2\, e^{-x^2}$.

5. (You should be able to evaluate special infinite series from various parts of Chapter 12.) Evaluate the following infinite series:

(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{\pi^{2n}}{36^n(2n)!}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(iii) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{2n+1}}{10^n}$

6. (You should know arclength, areas of surfaces of revolution, and parametric equations — Sections 9.1,9.2,11.1,11.2) A curve is specified by the parametric equations

$x = \ln t\ ,\ y = \dfrac{t^2}{2} + \ln t,\ 0 < t < \infty$

(i) At what values of t is the curve concave up?

(ii) At what Cartesian coordinates does the tangent line have slope equal to 5?

(iii) Set up, but do not evaluate, an integral for the length of the curve for $1 \leq t \leq 2$. You do not have to simplify your answer.

7. (You should know differential equations — Sections 10.1,10.3.) Find the solution to the differential equation that satisfies the initial condition.
$e^{x^2} y' = 2x(1+y), \qquad y(0) = 0.$

8. (You should know limits — Chapter 7, Section 12.1.) Determine the following limits.

(i) $\lim_{n\to\infty} a_n$, where $a_n = \ln(1+2n^2) - \ln(n^2)$.

(ii) $\lim_{x\to 0^+} (\cos 2x)^{1/x^2}$.