

INSTRUCTIONS:

(I) Start the solution to each problem at the top of a new page in the blue book.

(II) You must show work to get credit for an answer.

1. (15 points) Evaluate $\int \frac{dx}{x^3 + x}$.

2. (20 points) Evaluate $\int \frac{x^3}{\sqrt{1-x^2}} dx$.

3. (12 points) Write out the terms (not numeric values) in the trapezoid rule estimation (with $n = 6$) of $\int_2^4 \sin(\sqrt{x}) dx$.

4. (20 points) Evaluate $\int_1^\infty x e^{-2x} dx$.

5. (15 points)

(a) Give the general solution to the differential equation $y' = 2(1 + y^2)x$.

(b) What is the particular solution from (a) that, in addition, satisfies $y(0) = 1$?

6. (15 points) A curve is specified by the parametric equations $x = t^2 + 2t$ and $y = t^2 - 3t$.

(a) Give $\frac{dy}{dx}$ in terms of t .

(b) Give $\frac{d^2y}{dx^2}$ in terms of t .

7. (13 points) Set up BUT DO NOT EVALUATE the integral for the length of the curve in the first quadrant given parametrically by $x = 3 \cos(t)$ and $y = 5 \sin(t)$ from $(3, 0)$ to $(0, 5)$.

8. (25 points) Consider the two polar curves

(i) $r = 1 - \sin(\theta)$,

(ii) $r = 1$.

(a) Sketch the two curves.

(b) Find the area that is inside curve (i) but is outside curve (ii).

9. (15 points) Classify the following as to absolutely convergent, conditionally convergent or divergent:

(a) $\sum_{n=0}^{\infty} \frac{n^2 + 1}{2n^4 - 1}$,

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{2n^2}{18n^2 + 1}$.

10. (15 points) Find the interval of convergence of $\sum_{n=2}^{\infty} \frac{2^n x^n}{n-1}$.

11. (15 points) Find power series representations for

(a) $(1 + x^4)^{-2}$,

(b) $\int \frac{\sin(x)}{x} dx$.

12. (10 points) Evaluate $\sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^n$.

13. (10 points) How many terms are needed to estimate $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^5}$ to an error less than 10^{-10} ?