I. (72 points. 6 points each.) Write your answer to each of the following. If you do "work", show it, but no "work" is required.

(i) $\dfrac{d}{dx}(\tan^{-1} x) =$

(vii) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx =$

(ii) $\displaystyle\int \frac{1}{x}dx =$

(viii) $\displaystyle\int (\sec x)dx =$

(iii) $\displaystyle\int \left(\sum_{n=0}^{\infty} x^{2n}\right) dx =$

(ix) $\displaystyle\frac{d}{dx}\sum_{n=0}^{\infty} x^{2n} =$

(iv) $\displaystyle\lim_{n\to\infty} (1 + x/n)^n =$

(x) $\displaystyle\lim_{x\to\infty} \frac{(2x+1)^4}{7-25x^4} =$

(v) $\dfrac{d}{dx}(\tan x) =$

(xi) $\tan^{-1} \dfrac{1}{\sqrt{3}} =$

(vi) $\displaystyle\sum_{n=0}^{\infty} (\pi/4)^n =$

(xii) $\displaystyle\lim_{x\to 0} \frac{e^x + \cos x}{1 + \sin x} =$

II. (24 points. 12 points each.) Produce the indicated derivatives.

(i) $f(x) = x^{\cos^2 x}$, $\qquad f'(x) =$

(ii) $f(x) = \tan^{-1}[(\ln x)^3]$, $\qquad f'(x) =$

III. (18 points. 6 points each.) In each case state which substitution you would use to attempt the integration; i.e., $x = ?$ DO NOT PERFORM THE INTEGRATIONS!!!

1) $\int (9 - x^2)^{3/2} dx$

2) $\int \frac{x^2}{\sqrt{4x^2 - 25}} dx$

3) $\int \frac{dx}{x^{1/2} - x^{1/3}}$

IV. (28 points. 7 points each.) In each case give the first 4 non-zero terms in the Taylor Series Expansion about zero (i.e., the Maclaurin series expansion) for the function.

1) $f(x) = \frac{1}{1 - x}$

2) $f(x) = \sin x$

3) $f(x) = e^x$

4) $f(x) = (1 + x)^\pi$

V. (55 points. Each part valued as indicated.) In each case, perform the required integration. SHOW ANY WORK YOU DO!!!

1) (10 points.) $\int (\sin x)^7 (\cos x)^3 dx =$

2) (20 points.) $\int (2x+2)(\tan^{-1} x) dx =$

3) (10 points.) $\int \frac{g'(x)}{\sqrt{1-(g(x))^2}} dx =$

4) (15 points.) $\int \frac{5x^2+3x+1}{x^3+x} dx =$

VI. (24 points. 12 points each.) Evaluate the following limits. SHOW ANY WORK YOU DO! If L'Hospital's Rule is used, indicate with "L'H" exactly where you use it and verify that it is applicable. HINT: Use series in part 2).

1) $\lim_{x\to 0} \dfrac{e^x - \cos x}{\sin x} =$

2) $\lim_{x\to 0} \dfrac{x^{50}}{e^x - \displaystyle\sum_{n=0}^{49} \frac{x^n}{n!}} =$

VII. (20 points. 10 points each.) Determine whether each series converges or diverges; if it diverges to $+\infty$ (or to $-\infty$), say so. Give a "short" justification for your conclusion(s).

1) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{2 + \sin(n\pi)}\right)$

2) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{e^{2n}}{5^{2n}(2n)!}$

VIII. (36 points. Each part valued as indicated.)

1) (12 points.) Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} n! \left(\frac{x}{10}\right)^n$.

SHOW YOUR WORK!!

2) (24 points.) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^{3n}}{n8^n}$.

SHOW YOUR WORK!!

IX. (30 points. 10 points each.) A curve is given parametrically by $x = t + \cos t$ and $y = t + \sin t$ for $0 \leq t \leq 1$. Note that x and y are increasing functions of t and that the curve lies in the first quadrant.

1) $\dfrac{dy}{dx} =$

2) Give an integral (in terms of ONLY the variable t) which gives the length of the curve. DO NOT DO THE INTEGRATION!!!

3) Give an integral (in terms of ONLY the variable t) which gives the AREA of the surface obtained when the curve is rotated about the y-axis. DO NOT DO THE INTEGRATION!!

X. (40 points. 10 points each.) Let f be a function which has $n+1$ continuous derivatives on the interval between a and x. Then $f(x) = T_n(x) + R_n(x)$ where $T_n(x)$ is the Taylor Polynomial of degree n in the Taylor Series expansion of f about a and $R_n(x)$ is the remainder.

1)
$T_n(x) =$

2) Give the best bound you can on $|R_n(x)|$. That is, $|R_n(x)| \leq$?

Now suppose $f(x) = \dfrac{(x-2)^3}{6} - \dfrac{(x-2)^5}{60}$ and $a = 3$. DO THIS ON THE BACK OF PAGE 5.

3) $T_2(x) =$? (Don't "simplify" your answer!)

4) Using 2) above, set $x = 1$ and give the best bound you can on $|R_2(1)|$. That is,

$$|f(1) - T_2(1)| = |R_2(1)| \leq ?$$

XI. (24 points. 12 points each.) $\sum_{n=1}^{\infty} \frac{1}{n^{11/10}} = \sum_{n=1}^{N} \frac{1}{n^{11/10}} + R_N$. $\sum_{n=1}^{\infty} \frac{1}{n^{11/10}}$ is a convergent p-series.

1) Give the INTEGRAL upper bound on the remainder R_N. I.e., $|R_N| \leq$?

2) What is the smallest N for which you can guarantee that

$$\left| \sum_{n=1}^{\infty} \frac{1}{n^{11/10}} - \sum_{n=1}^{N} \frac{1}{n^{11/10}} \right| = |R_N| \leq \frac{1}{10}?$$

XII. (24 points.) The graph of the equation $\frac{(x-1)^2}{2^2} - \frac{(y+1)^2}{1^2} = 1$ is a hyperbola.

1) Give the coordinates of its vertices.
2) Give the coordinates of its foci.
3) Give the equations of its asymptotes.
4) Sketch the graph of the hyperbola. Put the vertices, foci, and asymptotes on your graph and identify them.

XIII. (35 points. Each part valued as indicated.)

1) (15 points.) Sketch the graph of the curve $r = 2 - \sin\theta$. Plot AT LEAST 8 points to get your graph. Use symmetry wherever possible. $\sqrt{3} \approx 1.732$ so $\sqrt{3}/2 \approx 0.87$ and $\sqrt{2} \approx 1.414$ so $\sqrt{2}/2 \approx 0.7$

2) (10 points.) Write down an integral (involving ONLY the variable θ) giving the area inside the curve. DO NOT DO THE INTEGRATION!!

3) (10 points.) Write down an integral (involving ONLY the variable θ) giving the length of the curve ($0 \leq \theta \leq 2\pi$ of course). DO NOT DO THE INTEGRATION!!