1. [72 pts.] Find each of the following. These are applications of facts listed as "Required Knowledge" on the Calc II web page - no justification is required.

(i) $\dfrac{d(\cos^{-1} x)}{dx} =$

(ii) $\dfrac{d(\ln x)}{dx} =$

(iii) $\dfrac{d\left(\sum_{n=1}^{\infty}(n^2 x^n)\right)}{dx} =$

(iv) $\lim_{x\to 5} e^x =$

(v) $\lim_{x\to\infty} \arctan x =$

(vi) $\lim_{n\to\infty} \dfrac{1-2n-3n^3}{2n^3-5} =$

(vii) $\int \dfrac{1}{x^2+1}\,dx = \qquad + C$

(viii) $\int 3^x dx = \qquad + C$

(ix) $\int \left(\sum_{n=0}^{\infty} n^2 x^n\right) dx = \qquad + C$

(x) $\sum_{n=1}^{\infty} (1/3)^n =$

(xi) $\tan^{-1}\sqrt{3} =$

(xii) $\lim_{n\to\infty} (1+2/n)^n =$

2. [40 pts.] Calculate the indicated derivatives: (You need not simplify.)

(i) $f(x) = x^\pi \cdot \pi^x$, $\qquad f'(x) =$

(ii) $g(x) = \arctan\left(\sum_{n=1}^{\infty} \dfrac{x^n}{n}\right)$, $\qquad g'(x) =$

(iii) $h(x) = x^x$, $\qquad h'(x) =$

(iv) $j(x) = \dfrac{\cos x}{\cosh x}$, $\qquad j'(x) =$

3. [40 pts.] Evaluate the following simplifying your answer to an explicit number.

(i) $\int_e^{e^3} 1/x \, dx$

(ii) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^{2n}(2n)!}$

(iii) $\sec(\sin^{-1}(1/3))$

(iv) $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!}$

4. [28 pts.] Evaluate the following limits - show the steps, use correct equations, and mark with L'H exactly where L'Hospital's Rule is used, after verification of its validity.

(i) $\lim_{x \to \ln 3} \frac{e^x - e^{-x} - 8/3}{x - \ln 3}$

(ii) $\lim_{t \to 0} (1 - \cos 3t)(\cot^2 t)$

5. [30 pts.] For this entire problem (remainder of page), let $f(x) = \ln(\sin^{-1} x)$.

(i) What is the natural domain of f? ________________________________

(ii) $f'(x) =$

(iii) Give rationale why f is one-to-one.

(iv) What is the domain of f^{-1}? ________________________________

(v) $f(1) =$

(vi) $f^{-1}(0) =$

6. [48 pts.] Evaluate the following integrals:

(i) $\displaystyle\int x^2 \sin x \; dx$

(ii) $\displaystyle\int \frac{4}{x^2(x^2+4)} \; dx$

(iii) $\displaystyle\int x^3(1-x^2)^{3/2} \; dx$

7. [30 pts.] Evaluate the following integrals:

(i) $\displaystyle\int_0^1 e^{x^2}\, dx$

(ii) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{|x|}}\, dx$

8. [12 pts.] Decide whether the sequence $\{a_n\}$ is increasing, decreasing or neither where $a_n = \dfrac{4^n (n!)^2}{(2n)!}$.

9. [30 pts.] Label the following series convergent or divergent - and sketch your justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^{3/2}}$ ______________________________

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n - \ln n}{n^2 + 10n + 1000}$ ______________________________

10. [30 pts.] (i) Determine the radius of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{n!}\, x^n$.

(ii) Determine the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^{2n}}{n2^n}$.

11. [25 pts.]

(i) Find a power series expansion for $\ln(1+x^4)$ about $a = 0$. Recall $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$. You need not calculate the interval of convergence for your answer.

(ii) Find the McLaurin series for $f(x) = (1+x)^{3/2}$, in power series expression and then writing the sum with the first 4 terms computed explicitly .

12. [20 pts.]

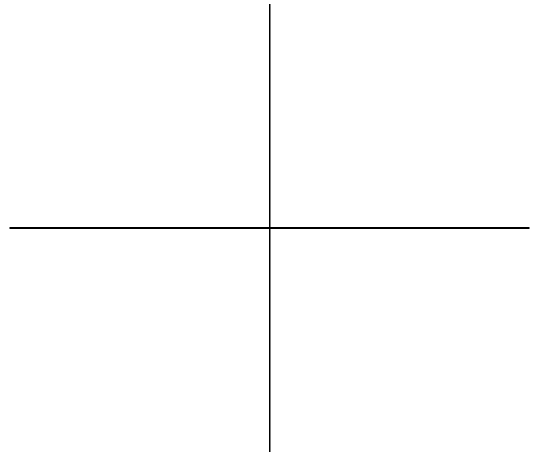
(i) Estimate $\cos(1)$ by using $T_4(1)$ - the fourth degree Taylor polynomial expanded at $a = 0$, evaluated at 1.

(ii) Give error bound in this estimate using alternating series. Make sure to show that the required conditions to apply the alternating series theory are satisfied.

13. [36 pts.] Consider the graph of points whose polar coordinates satisfy $r = \cos 2\theta$.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	1	0	-1	0	1	0	-1	0	1

Here is a sketch of the graph of this rose:



(i) Give a parametrization of this curve.

(ii) Find the slope of the line tangent to graph when $\theta = \dfrac{\pi}{4}$, $r = 0$. Show your work for credit.

(iii) Find the area enclosed by one leaf of this rose.

(iv) SET UP ONLY - the integral for the arclength of one leaf of the rose.

(v) SET UP ONLY - the integral for the surface area generated when the portion of the curve for $0 \leq \theta \leq \pi/4$ is revolved about the x-axis.

14. [12 pts.] (i) Give polar coordinates for the point whose rectangular coordinates are $(-1, 1)$.

(ii) Give rectangular coordinates for the point whose polar coordinates are $\left(2, \cos^{-1}\frac{1}{3}\right)$.

15. [22 pts.]

(i) Calculate the formula for the parabola of form $y = Ax^2 + Bx + C$ which goes through $(0, 1)$, $(1, 3)$, $(-1, 1)$.

(ii) Give the vertex, focus and directrix, and then sketch the graph.

vertex ____________________

focus ____________________

directrix ____________________

16. [15 pts.] Suppose f is a function with $f(1) = 1$, $f(2) = 2$ and $\lim_{n\to\infty} f(n) = 3$. Calculate $\sum_{n=1}^{\infty}\left(f(n+2) - \frac{1}{2}f(n+1) - \frac{1}{2}f(n)\right)$. [Hint: A partial sum telescopes.]