

Math 222 – Midterm – October 19, 1999

Put your name on your examination booklets.

You must show all your work; I will not give credit for answers without adequate justification.

Write everything, including scratch work, in the examination booklets. You do not have to do the problems in order, but be sure to identify your answers.

You may use simple calculators (without graphing or programming functions) although this should not be necessary. There is a short table of trig functions at the bottom of this sheet.

**Do not use calculators to replace expressions involving expressions like  $\pi$ , powers of  $e$ , square roots or logarithms with decimal approximations.**

- (1) [15] Choose *one* of the following numerical integration methods: Trapezoidal, Midpoint, or might.

(a) Write down the formula for this method when the function  $f(x)$  is integrated over the interval with endpoints  $a$  and  $b$ , using  $n$  subintervals. Explain the meanings of the different terms in your formula.

(b) Using the method you chose in part (a), find a value of  $n$  that will ensure that the error in approximating  $\int_1^3 \ln x \, dx$  is no greater than  $1/100$ . Hint: The error estimate is one of the following:

$$|E| \leq \frac{K(b-a)^3}{24n^2} \quad \text{or} \quad |E| \leq \frac{K(b-a)^5}{180n^4} \quad \text{or} \quad |E| \leq \frac{K(b-a)^3}{12n^2}.$$

- (2) [24] Determine whether each series converges or diverges, and explain your answer. Find the sum of *one* of the convergent series.

(a)  $\sum_{n=1}^{\infty} \frac{1}{1+100n^2}$

(b)  $\sum_{n=1}^{\infty} \frac{n}{1+100n^2}$

(c)  $\sum_{n=1}^{\infty} \frac{n^2}{1+100n^2}$

(d)  $\sum_{n=1}^{\infty} \frac{2^{n-1}3^{n-1}5}{7^{n-1}}$

- (3) [15] Determine the following limits:

(a)  $\lim_{n \rightarrow \infty} \frac{n!}{3 \cdot 5 \cdot 7 \cdots (2n+1)}$

(b)  $\lim_{x \rightarrow \infty} x \left( \frac{\pi}{2} - \tan^{-1} x \right)$

Hint:  $\frac{k}{2k+1} \leq \frac{1}{2}$ .

- (4) [10] Consider the curve  $y = e^x$  for  $0 \leq x \leq 1$ . Set up, but do not evaluate, integrals for each of the following:
- The arc length of this curve, using  $x$  as the independent variable.
  - The arc length of this curve, using  $y$  as the independent variable.
  - The area of the surface generated by rotating this curve around the  $X$ -axis, using  $x$  as the independent variable.
  - The area of the surface generated by rotating this curve around the  $Y$ -axis, using  $x$  as the independent variable.

- (5) [13] Evaluate *one* of the following integrals:

(a)  $\int_0^{\pi} x^2 \sin x \, dx$

(b)  $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \, dx$

- (6) [13] Evaluate *one* of the following integrals:

(a)  $\int \frac{dx}{(x^2 + 4)^{\frac{3}{2}}}$

(b)  $\int \frac{dx}{(x - 2)(x + 1)}$

- (7) [10] Consider the integrals  $\int_{-1}^1 x^{-3} \, dx$  and  $\int_{-1}^1 x^{-\frac{1}{3}} \, dx$ . One of these makes no sense; the other does. Explain, and evaluate the one which does make sense.

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\infty$	0