

Math 222 – Test 2 – November 17, 1999

Put your name on your examination booklets.

You must show all your work; I will not give credit for answers without adequate justification.

Write everything, including scratch work, in the examination booklets. You do not have to do the problems in order, but be sure to identify your answers.

You may use simple calculators (without graphing or programming functions) although you should not need to. There is a short table of trig functions at the bottom of this sheet.

**Do not use calculators to replace expressions involving expressions like  $\pi$ , powers of  $e$ , square roots or logarithms with decimal approximations.**

- (1) [30] Determine whether each of the following converges absolutely, converges conditionally, or diverges.

(a) 
$$\sum_{n=0}^{\infty} (-1)^n \frac{n!(n+1)!}{(2n)!}$$

(b) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{100n+1}$$

(c) 
$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{n}-1}$$

- (2) [10] Find the radius of convergence of 
$$\sum_{n=0}^{\infty} \frac{x^{2n}}{1 \cdot 5 \cdot 9 \cdots (4n+1)}.$$

- (3) [10] Find the interval of convergence of 
$$\sum_{n=1}^{\infty} \frac{x^n}{n2^n}.$$

- (4) [10] Find the first three non-zero terms in the Taylor series for  $x^{\frac{3}{2}}$  centered at  $a = 1$ .

- (5) [10] Find a series representation for  $\int_0^1 \cos \sqrt{x} \, dx$ .

- (6) [15]

- (a) Suppose the first 5 terms in the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$  are used to approximate the sum of the series. What is the best estimate you can give for the remainder (in absolute value)?

- (b) Now answer the same question if the series is replaced with  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ .

- (7) [15] Suppose the Maclaurin series is used to estimate the value of  $e^x$  for  $|x| \leq 1$ . How many terms should be used to guarantee an error of at most  $10^{-3}$ ? You may use the approximation  $e \leq 3$ . [Hint: What is the  $(n+1)^{\text{th}}$  derivative of  $e^x$ ?]