Mathematical Sciences 222 - 02
Calculus II
Fall 2003
Second Exam
25 November

SHOW YOUR WORK — SHOW YOUR WORK

NO CALCULATORS! — NO CALCULATORS!

Total value 150 points. Each part valued as indicated.

I. (17 points. Each part valued as indicated.) In each case write out the first 4 non-zero terms of the Taylor Series Expansion about 0 (i.e. the usual expansion−−−the Maclaurin Series Expansion) for the function, and give R, the radius of convergence of the series. No "work" is necessary.

1) (3 points) $e^x = ?$

2) (3 points) $\sin x = ?$

3) (3 points) $\frac{1}{1-x} = ?$

4) (4 points) $\sqrt{1+x} = ?$

5) (4 points) $x^2 e^{x^2} = ?$

II. (24 points. 6 points each.) In each case give a **simple** answer for the sum of the series.

1) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$

2) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(\frac{\pi}{2})^{2n}}{(2n)!}$

3) $\displaystyle\sum_{n=1}^{\infty} \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$

4) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{n!}$

III. (20 points. 5 points each.) Produce the following limits. Show your work!! (NO WORK = NO POINTS)

1) $\lim_{n\to\infty} \frac{(2n+1)^3}{4n^3+7n+800} = ?$

2) $\lim_{n\to\infty} \left(\frac{n-5}{n}\right)^n = ?$

3) $\lim_{n\to\infty} (\log n - \log(n+\sqrt{n})) = ?$

4) $\lim_{n\to\infty} (n+\log n)^{1/n} = ?$

IV. (18 points.) In each case tell whether the series converges or diverges and WHY. (WRONG REASON = NO POINTS)

1) $\sum_{n=0}^{\infty} (-1)^n \left(\frac{n}{10n+1000}\right)$

2) $\sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{(n!)^2}$

3) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$

V. (20 points.) Give the set of x's for which the following series converges. SHOW YOUR WORK!

$$\sum_{n=1}^{\infty} (-1)^n n \left(\frac{2x-3}{5}\right)^n$$

VI. (12 points.) $S = \sum_{n=0}^{\infty}(-1)^n \frac{1}{n!} = S_N + R_N$ where $S_N = \sum_{n=0}^{N}(-1)^n \frac{1}{n!}$. What is the smallest N for which you can guarantee that $|S - S_N| \leq \frac{1}{500}$?

VII. (22 points. Each part valued as indicated.)

$$f(x) = \int_0^{x^2} t(\sin t^2)dt.$$

1) (8 points.) Give the Taylor Series Expansion ($a = 0$) for $f(x)$.

2) (2 points.) For what values of x is the expansion valid?

3) (12 points.) What is the minimum number of terms in the TS Expansion of f needed to approximate $f(1)$ to within $\frac{1}{5{,}000}$? Why? What is the approximation (written as a sum of fractions)? SHOW YOUR WORK!