I. (46 points. Each part valued as indicated.) In each case write out the first 4 non-zero terms of the Taylor Series Expansion about 0 (i.e. the usual expansion–––the Maclaurin Series Expansion) for the function, and give R, the radius of convergence of the series. Do only "necessary" work.

1) (6 points) $\frac{1}{1-x} =$

2) (6 points) $\cos x =$

3) (6 points) $e^x =$

4) (8 points) $(1+x)^{3/2} =$

5) (10 points) $\frac{x^3}{1+\frac{x}{2}} =$

6) (10 points) $\frac{d}{dx}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n =$

II. (30 points. 10 points each.) Sum each of the following series. Give each answer in as simple a form as possible.

1) $\sum_{n=3}^{\infty}\left(-\frac{1}{2}\right)^n =$

2) $\sum_{n=0}^{\infty}(-1)^n\frac{(\frac{\pi}{2})^{2n+1}}{(2n+1)!} =$

3) $\sum_{n=3}^{\infty}\left[\frac{1}{n}-\frac{1}{(n+1)}\right] =$

III. (16 points. 8 points each.) Produce the following limits. If the answer requires work, SHOW YOUR WORK!

1) $\lim_{k\to\infty}\left(\frac{k+e}{k}\right)^k =$

2) $\lim_{n\to\infty}(\ln(n\pi+1)-\ln(n-\sqrt{n})) =$

IV. (34 points.) In each case tell whether the series converges or diverges and WHY. (WRONG REASON = NO POINTS) If it converges because of the integral test, DO THE INTEGRAL TEST!

1) $\sum_{n=0}^{\infty}\frac{1}{n\sqrt{n}}$

2) $\sum_{n=0}^{\infty}(-1)^n\frac{(\pi^2+7n-10n^2)^2}{(5n+13)^4}$

3) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)}$

4) $\sum_{k=1}^{\infty}\frac{k^2+\pi}{2k^4-25}$

V. (20 points.) Give the set of x's for which the following series converges. SHOW YOUR WORK!

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left(\frac{5-2x}{7} \right)^n$$

VI. (12 points.) $S = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} = S_N + R_N$ where $S_N = \sum_{n=0}^{N} (-1)^n \frac{1}{(2n)!}$. What is the smallest N for which you can guarantee that $|S - S_N| = |R_N| \leq \frac{1}{500}$?

VII. (28 points.) Let $f(x) = (1+x)\ln(1+x)$ and let $T_n(x)$ be the n-th degree Taylor Series polynomial in the Taylor Series expansion of f about zero (i.e., $a = 0$). DO YOUR WORK ON THE BACK OF P-2.

(1) (14 points.) What is the smallest n for which you can guarantee that

$$\left| f\left(\frac{1}{4}\right) - T_n\left(\frac{1}{4}\right) \right| \leq \frac{1}{40}?$$

(2) (14 points.) For that n, write out $T_n(\frac{1}{4})$ as a sum of fractions.

VIII. (20 points.) Let $f(x) = \dfrac{1}{\sqrt{2 - x^3}}$.

1) (10 points.) Give the infinite Taylor Series expansion about 0 (i.e. the Maclaurin Series expansion) for $f(x)$. <u>THINK</u>!

2) (5 points.) Give R, the radius of convergence, of your series in part 1).

3) (5 points.) Give $T_8(x)$ for this particular case.

IX. (12 points. 6 points each.) In each case tell whether the statement is true or false. If false, give a counterexample.

(1) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(2) If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} (a_n + b_n)$ diverges.