SHOW YOUR WORK | Math 222-02 | SHOW YOUR WORK

Calculus II

Spring 2000

Exam #3

NO CALCULATORS | 4 May | NO CALCULATORS

Name ______________________________

Total value 75 points. Each part valued as marked.

I. (10 points. 2 points each.) Write down the first four non-zero terms in the infinite Taylor series expansion (about $x = 0$) for each of the following:

1) $f(x) = \dfrac{1}{1-x} = ?$

2) $f(x) = \cos x = ?$

3) $f(x) = e^x = ?$

4) $f(x) = (1 + x^2)^\pi = ?$

5) $f(x) = x^2 e^{-x^2} = ?$

II. (10 points) Derive/Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n(4^n)}$.

SHOW YOUR WORK!

III. (10 points. 2 1/2 points each.) In each case state whether the series converges or diverges and give a reason/argument. If your reason/argument is wrong, you will get no part credit.

1) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(\ln k)^2}$

2) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}+100}{k^2-18}$

3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(1-\frac{1}{n}\right)^n$

4) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \left(\frac{2+\cos(k\pi)}{k}\right)$

IV. (10 points.) Write down the finite Taylor Series expansion about $x = -1$ for the function $f(x) = x^5$. Use $n = 3$; i.e., give the first four terms (whether zero or not) PLUS an explicit remainder term.

V. (10 points.) Derive/Find the interval of convergence of the power series $\sum_{k=2}^{\infty} \frac{(-1)^k(3x-1)^k}{4^k(\ln k)}$.

SHOW YOUR WORK!

VI. (7 points.) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n!)^2 x^{2n+1}}{(2n)!}$.

SHOW YOUR WORK!

VII. (8 points.) Let $S = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$. Find two numbers A & B which differ by at most $\frac{1}{100}$ (i.e. such that $|A - B| \le \frac{1}{100}$) such that $A \le S \le B$. SHOW YOUR WORK!

VIII. (10 points.) Let $S = \sum_{k=1}^{\infty} \frac{1}{k^3}$ and $S_n = \sum_{k=1}^{n} \frac{1}{k^3}$.

1) Use integral test arguments to bound the error in using S_N to estimate S. I.e., $|S - S_n| \le$ what bound? SHOW YOUR WORK!

2) Using this argument, what is the smallest N for which you can guarantee that $|S - S_N| \le \frac{1}{100}$? SHOW YOUR WORK!