

SHOW YOUR WORKSHOW YOUR WORKNO CALCULATORS!NO CALCULATORS!Total value 100 points. Each part valued as indicated. $\log = \ln$

I. (12 points. 2 points per part.) In each case write out the first 4 non-zero terms of the Taylor Series Expansion about 0 (i.e. the usual expansion---the Maclaurin Series Expansion) for the function, and give R , the radius of convergence of the series. No "work" is necessary.

1) $e^x = ?$

2) $\sin x = ?$

3) $\cos x = ?$

4) $\frac{1}{1-x} = ?$

5) $(1+x)^{1/3} = ?$

6) $\int xe^x dx = ?$

II. (16 points. 4 points each.) In each case give a **simple** answer for the sum of the series.

1) $\sum_{n=2}^{\infty} \left(-\frac{3}{4}\right)^n$

2) $\sum_{n=0}^{\infty} (-1)^n \frac{(3\pi)^{2n}}{2^{2n}(2n)!}$

3) $\sum_{n=2}^{\infty} \left[\frac{1}{(\ln n)^2} - \frac{1}{(\ln(n+1))^2} \right]$

$$4) \sum_{n=0}^{\infty} \frac{(-1)^n e^n}{n!}$$

III. (12 points. 3 points each.) Produce the following limits.

$$1) \lim_{x \rightarrow \infty} \frac{(1 + x + 2x^2)^2}{(x + 10)^4} = ?$$

$$2) \lim_{x \rightarrow \infty} \left(\frac{x - \ln 2}{x} \right) = ?$$

$$3) \lim_{n \rightarrow \infty} (\log(2n + \sqrt{n}) - \log n) = ?$$

$$4) \lim_{x \rightarrow \infty} x^{1/\log x} = ?$$

IV. (14 points.) In each case tell whether the series converges or diverges and WHY. (WRONG REASON = NO POINTS)

$$1) \sum_{n=2}^{\infty} \frac{1}{n(\log n)(\log \log n)}$$

$$2) \sum_{n=0}^{\infty} (-1)^n \left(\frac{(\log n) + n^2}{10n^2 + 1000} \right)$$

$$3) \sum_{n=0}^{\infty} (-1)^n \frac{20n^3}{n!}$$

V. (14 points.) Give the set of x 's for which the following series converges. SHOW YOUR WORK!

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{3 - 2x}{7} \right)^n$$

VI. (7 points.) $S = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} = S_N + R_N$ where $S_N = \sum_{n=0}^N (-1)^n \frac{1}{(2n)!}$. What is the smallest N for which you can guarantee that $|S - S_N| \leq \frac{1}{500}$?

VII. (14 points. Each part valued as indicated.)

$$f(x) = \int_0^x \frac{t^5}{1-t^2} dt$$

1) (6 points.) Give the Taylor Series Expansion (about $a = 0$) for $f(x)$.

2) (2 points.) For what values of x is the expansion valid?

3) (3 points.) $f^{(300)}(0) = ?$

4) (3 points) $f^{(275)}(0) = ?$