Abnormal and Pronormal Subgroups of a Direct Product of Groups

by Ben Brewster

While the subgroups of a direct product are well-understood, it has been a project to characterize subgroups of a direct product which satisfy some extra properties. It is an exercise in group theory to show that if $U \leq A \times B = G$, then U is normal in G if and only if $\pi_X(U)$, $U \cap X$ are normal in X for $X = A, B$ and $\pi_X(U)/U \cap X \leq Z(X/U \cap X)$.

In general, a subgroup U is pronormal in G provided for each $g \in G$, there is $x \in \langle U, U^g \rangle$ such that $U^x = U^g$. U is abnormal in G provided for each $g \in G$, $g \in \langle U, U^g \rangle$. The normalizer of a pronormal subgroup in G is abnormal in G.

Results obtained recently in joint effort with A. Martinez-Pastor and M.D. Perez-Ramos characterize the pronormal and abnormal subgroups of a direct product of groups. I hope to present the results and some of the more accessible proofs.