

TITLE: The non-abelian tensor product of finite groups is finite: a homology free proof.

ABSTRACT: R. Brown and J. L. Loday first introduced the non-abelian tensor product $G \otimes H$ for groups G and H in context with an application in homotopy theory. Let G and H be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions of G and H are said to be compatible, if ${}^h g h' = {}^{hgh^{-1}} h'$ and ${}^g h g' = {}^{ghg^{-1}} g'$ for all $g, g' \in G, h, h' \in H$. The non-abelian tensor product $G \otimes H$ is defined provided G and H act compatibly. In such a case $G \otimes H$ is the group generated by the symbols $g \otimes h$ with relations $gg' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$, where ${}^g g' = gg'g^{-1}$ and ${}^h h' = hh'h^{-1}$. In their 1987 paper, *Some computations of non-abelian tensor products of groups*, Brown, Johnson and Robertson mention eight open problems. The first problem is phrased as follows: Let G and H be finite groups acting compatibly on each other. Then is it true that $G \otimes H$ is finite? In the same year, G. J. Ellis answered the question in the affirmative using homological methods. Brown, Johnson and Robertson add that no purely algebraic proof is known. In this talk I will present a homology free and purely group theoretic proof that the non-abelian tensor product of two finite groups is finite.