Incorporating parameter uncertainty into the setup of EWMA control charts monitoring normal mean

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Most of the literature concerned with the design of control charts relies on perfect knowledge of the distribution for at least the good (so-called in-control) process. In the simplest case, monitoring normal mean, one usually assumes that the in-control mean $\mu_0$ and the variance $\sigma$ (typically assumed to be fix) are known. Then the standard EWMA looks like this:

\[
Z_0 = z_0 = \mu_0 = 0, \\
Z_i = (1 - \lambda)Z_{i-1} + \lambda X_i, \ i = 1, 2, \ldots, \\
L = \inf \left\{ i \in \mathbb{N} : |Z_i - \mu_0| > c_E \sqrt{\frac{\lambda}{2 - \lambda}} \right\}.
\]

The parameters $\lambda \in (0, 1]$ and $c_E > 0$ are chosen to enable a certain useful detection performance (not too much false alarms and quick detection of changes). The most popular performance measure is the so-called Average Run Length (ARL), that is $E_{\mu}(L)$ for the true mean $\mu$. If the distribution parameters, $\mu_0$ and $\sigma$, have to be estimated by sampling data during a pre-run phase, then these uncertain parameters effect, of course, the behavior of the applied control chart. Typically the ARL is increased. Most of the papers about characterizing the uncertainty impact deal with the changed ARL patterns and possible adjustments. Here, a different way of designing the chart is treated: Setup the chart through specifying a certain false alarm probability such as $P_{\mu_0}(L \leq 1000) \leq \alpha$. This results in a specific $c_E$. Here we describe a feasible way to determine this value $c_E$ also in case of unknown parameters for a pre-run series of size $N$. Moreover, some related results for monitoring normal variance based on a sequences of batches of size $n$ are presented.

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