

Signature defects, cusp singularities, and special values of L -functions

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Let P be an isolated normal singularity of an n -dimensional complex space. Hirzebruch showed that under certain circumstances one can define a rational number $\delta(P)$, called the signature defect, that measures the failure of the signature theorem applied to a compact neighborhood U of P . In the special case that P arises from the minimal compactification of a Hilbert modular surface, Hirzebruch showed that $\delta(P)$ equals the special value at $s = 1$ of a certain L -function attached to a real quadratic field. He conjectured that a similar statement should hold for all Hilbert modular varieties; this was proved in the 80s by Atiyah-Donnelly-Singer and Müller. Their proofs used the Atiyah-Patodi-Singer index theorem applied to the manifold with boundary obtained by slicing P away from U .

In this talk we discuss a generalization of Hirzebruch's conjecture, due to Satake, that applies to all the special values of the above L -functions, and we discuss its proof. Instead of the APS index theorem, we use the explicit combinatorics of a toroidal resolution of P . This has the advantage that one can see directly how the intersection numbers arising in the computation of $\delta(P)$ contribute to the special values.

This is joint work with R. Sczech and J. Sturm.