Topological Invariants of Translation Actions

Let $\partial_\infty \mathbf{R}^m$ denote the "sphere at infinity" of $\mathbf{R}^m$ (ie. $\partial_\infty \mathbf{R}^m$ is the set of asymptotic equivalence classes of geodesic rays $\gamma : [0, \infty) \to \mathbf{R}^m$). There are two notions of a "neighborhood" in $\mathbf{R}^m$ of $e \in \partial_\infty \mathbf{R}^m$: (1) half-spaces in $\mathbf{R}^m$ "perpendicular" to e and (2) the intersection with $\mathbf{R}^m$ of ordinary neighborhoods of e in the (compact) space $\mathbf{R}^m \cup \partial_\infty \mathbf{R}^m$.

Let G be a group, n be a non-negative integer, and $\rho : G \to \mathrm{Transl}(\mathbf{R}^m)$ be an action by translations of G on $\mathbf{R}^m$. The "Bieri-Neumann-Strebel-Renz invariants" $\Sigma^n(\rho)$ can be defined using a topological property of ρ called *controlled* $(n-1)$*-connected* (or CC^{n-1}) *in the direction* $e \in \partial_\infty \mathbf{R}^m$. This will be explained during the talk. This property is defined using notion (1) of neighborhood of e.

There is a natural definition competing with CC^{n-1} which uses notion (2). This leads to the idea of the action ρ being *bounded* $(n-1)$*-connected* (or BC^{n-1}) *in the direction* e. A fundamental question is: how are these properties CC^{n-1} and BC^{n-1} related? For cocompact actions, one relation is the following: ρ is BC^{n-1} in the direction e if and only if ρ is CC^{n-1} in all the directions lying in an open $\frac{\pi}{2}$-neighborhood of e. The proof of this statement will be sketched for the case $n = 1$.