Homework 2
Math 518
October 23, 2006

1. Show that any map $S^4 \to S^2 \times S^2$ induces the zero map on $H^4(-;\mathbb{Z})$.

2. Use cup products to compute the map $H^*(\mathbb{CP}^n;\mathbb{Z}) \to H^*(\mathbb{CP}^n;\mathbb{Z})$ induced by the map $\mathbb{CP}^n \to \mathbb{CP}^n$ that is the quotient of the map $\mathbb{C}^{n+1} \to \mathbb{C}^{n+1}$ raising each coordinate to the d^{th} power, $(z_0,\ldots,z_n) \mapsto (z_0^d,\ldots,z_n^d)$, for a fixed integer d. You might want to consider the case $n=1$ first.

3. If X and Y are both finite CW-complexes, what is the relationship between $\chi(X), \chi(Y)$, and $\chi(X \times Y)$?

4. Let m and n be positive integers, $X = S^m \times S^n$, and $Y = S^m \vee S^n \vee S^{n+m}$. Show that $H_q(X;G) \cong H_q(Y;G)$ and $H^q(X;G) \cong H^q(Y;G)$ for any abelian group G and any integer q. Then show that X and Y are *not* homotopy equivalent.

5. (a) Given a finite CW-complex X, calculate the reduced cohomology groups of ΣX (reduced suspension of X) in terms of the reduced cohomology groups of X.
   (b) Determine the ring structure on the cohomology of ΣX.

6. Assuming as known the cup product structure on the torus $S^1 \times S^1$, compute the cup product structure in $H^*(M_g;\mathbb{Z})$ for M_g the closed orientable surface of genus g by using the quotient map from M_g to a wedge sum of g tori. (There is a picture in Hatcher page 228)