Homework 2
Math 518
October 23, 2006

1. Show that $M \times N$ is orientable if and only if both M and N are orientable.

2. Show that if M is a compact, closed, connected nonorientable 3-manifold, then $\pi_1(M)$ is not a finite group. (Hint: Show that $H_1(M;\mathbb{Z})$ cannot be a finite group.)

3. Show that if M is a compact, closed, connected, orientable n-manifold with $H_1(M;\mathbb{Z}) = 0$, then no nonorientable compact, closed $(n-1)$-manifold A can be embedded in M.

4. If M and N are two oriented, compact, closed, connected n-manifolds, construct their connected sum $M\#N$ by taking a nicely embedded n-disk in each, removing its interior, and gluing the remainders together via an orientation *reversing* homeomorphism on the boundary spheres of these disks. Show that the cohomology ring of $M\#N$ is isomorphic to the direct product of the cohomology rings for M and N with unity elements identified (ie, $1 \in H^0(M;R)$ is identified with $1 \in H^0(N;R)$) and orientation classes identified. Similarly, the multiples of these identifications must also be made. In particular, however, the cup product of positive dimensional classes, one from each manifold, is zero.

5. Show that if M is a compact R-orientable n-manifold, then the boundary map $H_n(M, \partial M; R) \to H_{n-1}(\partial M; R)$ sends a fundamental class for $(M, \partial M)$ to a fundamental class for ∂M.

6. Prove there is no homotopy equivalence $f : \mathbb{CP}^{2n} \to \mathbb{CP}^{2n}$ that reverses orientation, ie induces multiplication by -1 on $H_{4n}(\mathbb{CP}^{2n};\mathbb{Z})$.