# Quiz 1, Calc 2, Section 3

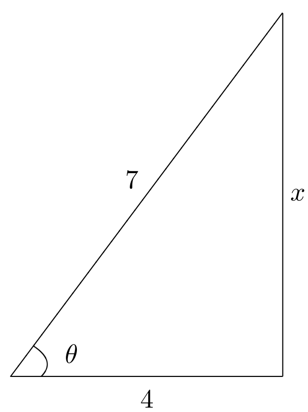
January 29 2007

1.

(a) Find $\tan(\cos^{-1}(4/7))$.

(b) Find $\sin(\tan^{-1}(-3))$.

*Solution*: (a) We want to get a numerical value for $\tan(\cos^{-1}(4/7)$, so we start with the following. Let $\theta = \cos^{-1}(4/7)$, so $\cos\theta = 4/7$. Now inverse cosine will only give an angle in the interval $[0, \pi]$, so $0 \leq \theta \leq \pi$. Since $\cos\theta = 4/7 > 0$, we know that $0 \leq \theta \leq \pi/2$. Now we can draw our triangle.

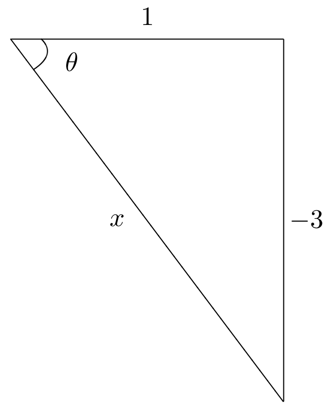



From the Pythagorean theorem, we can calculate that $4^2 + x^2 = 7^2$, or after simplifying $x^2 = 49 - 16 = 33$. So $x = \sqrt{33}$. From the triangle, we see thattan $\theta = \sqrt{33}/4$. so we have

$$\tan(\cos^{-1}(4/7)) = \frac{\sqrt{33}}{4}.$$

(b) For this one, we use the same method as in part (a). Let $\theta = \tan^{-1}(-3)$, so $\tan\theta = -3$. Inverse tangent gives an angle in the interval $[-\pi/2, \pi/2]$. Since

$\tan\theta = -3 < 0$, we know that $-\pi/2 < \theta < 0$, so we can draw our triangle.




Using the Pythagorean theorem, we get $1^2 + (-3)^2 = x^2$, or $x^2 = 10$. So $x = \sqrt{10}$. From the triangle, we see that $\sin\theta = -3/\sqrt{10}$, so we have

$$\sin(\tan^{-1}(-3)) = \frac{-3}{\sqrt{10}}.$$

2. Find $y'$ for

(a) $y = \sin^{-1}(\sqrt{x+1})$.

(b) $y = \cot^{-1}(x^2)$.

*Solution*: (a) This is a chain rule situation. So to take the derivative, we have to work from the outside in . This gives

$$\begin{aligned}
y &= \sin^{-1}(\sqrt{x+1}) \\
\frac{dy}{dx} &= \frac{d}{dx}(sin^{-1}(\sqrt{x+1})) \\
&= \frac{1}{\sqrt{1-(\sqrt{x+1})^2}}\frac{d}{dx})(\sqrt{x+1}) \\
&= \frac{1}{\sqrt{1-(x+1)}}\frac{1}{2\sqrt{x+1}} \\
&= \frac{1}{2\sqrt{-x}\sqrt{x+1}} \\
&= \frac{1}{2\sqrt{-x^2-x}}.
\end{aligned}$$

So

$$y' = \frac{1}{2\sqrt{-x^2-x}}.$$

(b) This is another chain rule situation. In this case we get

$$\begin{aligned} y &= \cot^{-1}(x^2) \\ \frac{dy}{dx} &= \frac{d}{dx}(\cot^{-1}(x^2)) \\ &= \frac{-1}{1+(x^2)^2}\frac{d}{dx}(x^2) \\ &= \frac{-1}{1+x^4}2x \\ &= \frac{-2x}{1+x^4}. \end{aligned}$$

So

$$y' = \frac{-2x}{1+x^4}.$$

3. Evaluate

$$\int \frac{2}{8x-3}dx$$

*Solution*: To evaluate this integral we need to make a substitution. Let $u = 8x - 3$. Then $du = 8dx$, or $(1/8)du = dx$. This gives

$$\begin{aligned} \int \frac{2}{8x-3}dx &= \int \frac{2}{u}\frac{1}{8}du \\ &= \frac{2}{8}\int \frac{1}{u}du \\ &= \frac{1}{4}\ln|u| + C \\ &= \frac{1}{4}\ln|8x-3| + C. \end{aligned}$$