Quiz 2, Calc 2, Section 3

February 5, 2007

1.Find y' where

(a) $y = \log_5(x^2 + 4x)$

(b) $y = e^{\tan(3x)}$

Solution: (a) This is a chain rule situation. The first step which might simplify the problem is to rewrite y as follows,

$$y = \log_5(x^2 + 4x) = \frac{\ln(x^2 + 4x)}{\ln 5}.$$

Using this rewritten form of y, we get

$$\begin{aligned} y' &= \frac{d}{dx}\left(\frac{\ln(x^2 + 4x)}{\ln 5}\right) \\ &= \frac{1}{\ln 5}\frac{1}{x^2 + 4x}\frac{d}{dx}(x^2 + 4x) \\ &= \frac{2x + 4}{(x^2 + 4x)\ln 5}. \end{aligned}$$

(b) Again, this is a chain rule situation. The chain rule tells us to take the derivative from the outside in, so we get

$$\begin{aligned} y' &= \frac{d}{dx}\left(e^{\tan(3x)}\right) \\ &= e^{\tan(3x)}\frac{d}{dx}(\tan(3x)) \\ &= e^{\tan(3x)}\sec^2(3x)3 \\ &= 3\sec^2(3x)e^{\tan(3x)}. \end{aligned}$$

2. Evaluate the following integrals

(a) $\int x^2 2^{x^3}\ dx.$

(b) $\int \cot x\ dx.$

Solution: (a) This integral looks like a prime candidate for a substitution. Let $u = x^3$, and so $du = 3x^2$. Or $(1/3)du = x^2 dx$. Using this substitution, we can rewrite the integral

$$\begin{aligned}\int x^2 2^{x^3}\ dx &= \frac{1}{3}\int 2^u\ du \\ &= \frac{1}{3}\frac{2^u}{\ln 2} + C \\ &= \frac{1}{3\ln 2}2^{x^3} + C.\end{aligned}$$

Another way to approach this integral is to write it in terms of e and solving it that way. If we do that, then we get

$$\int x^2 2^{x^3}\ dx = \int x^2 e^{x^3 \ln 2}\ dx.$$

Here, we can use the substitution $u = x^3 \ln 2$, so $du = 3x^2 \ln 2\ dx$. Rewritten we have $(1/3\ln 2)du = x^2\ dx$. Using this substitution, we get

$$\begin{aligned}\int x^2 e^{x^3 \ln 2}\ dx &= \frac{1}{3\ln 2}\int e^u\ du \\ &= \frac{1}{3\ln 2}e^u + C \\ &= \frac{1}{3\ln 2}e^{x^3 \ln 2} + C.\end{aligned}$$

(b) This probelm becomes a lot easier when we rewrite $\cot x$ in terms of $\sin x$ and $\cos x$.

$$\int \cot s\ dx = \int \frac{\cos x}{\sin x}\ dx.$$

From here, we can use the substitution $u = \sin x$, $du = \cos x\ dx$. This gives

$$\begin{aligned}\int \cot x\ dx &= \int \frac{\cos x}{\sin x}\, dx \\ &= \int \frac{1}{u}\, du \\ &= \ln|u| + C \\ &= \ln|\sin x| + C.\end{aligned}$$