Quiz 3, Calc 2, Section 3

February 12, 2007

1. The half-life of the element Californium is 898 years. How much will be left of 24 grams of Californium after 2694 years?

Solution: There are a couple of ways to solve this problem. I'm going to do it the way you would solve any general problem like this. We know that this is an exponential decay problem, so if y is the number of grams of Californium, then we have

$$y = y_0 e^{kt}.$$

In the equation, y_0 is the initial amount of Californium we started with (at time 0), and k is the decay constant. We already know that $y_0 = 24$ from the question since we start with 24 grams of Californium. We need to figure out what k is. To do this we use the half-life. We know that after 898 years, half of the Californium will be gone, so after 898 years we have

$$\begin{aligned} 12 &= 24e^{k898} \\ \frac{1}{2} &= e^{898k} \\ \ln\left(\frac{1}{2}\right) &= 898k \\ \frac{1}{898}\ln\left(\frac{1}{2}\right) &= k. \end{aligned}$$

Now that we have k, we can figure out how much of the Californium is left after 2694 years. We just plug in the values for y_0 and k into our formula and set $t = 2694$.

$$y = 24e^{\frac{2694}{898}\ln(1/2)}.$$

You can simplify this a little more since $2694 = 898 * 3$.So we have

$$\begin{aligned} y &= 24e^{3\ln(1/2)} \\ &= 24e^{\ln((1/2)^3)} \\ &= 24\left(\frac{1}{2}\right)^3 \\ &= 3. \end{aligned}$$

After 2694 years there will be 3 grams of Californium left.

2. Evaluate the following limits

(a) $\displaystyle\lim_{x\to 0}\frac{\tan x - x}{x^2}.$

(b) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$.

(c) $\lim_{x\to 1} \frac{x^3 + x^2 - 37x + 35}{x^3 - 2x^2 - 5x + 6}$.

Solution: (a) This limit goes tp $\frac{0}{0}$, so we can use L'Hopitals rule to get

$$\lim_{x\to 0} \frac{\tan x - x}{x^2} \stackrel{L'H}{=} \lim_{x\to 0} \frac{\sec^2 x - 1}{2x}.$$

Now the new limit on the right is again going to $\frac{0}{0}$ since $\sec 0 = 1$. So we have to use L'Hopitals rule again to get

$$\lim_{x\to 0} \frac{\sec^2 x - 1}{2x} \stackrel{L'H}{=} \lim_{x\to 0} \frac{2\sec x \sec x \tan x}{2} = \frac{0}{2} = 0.$$

(b) This limit goes to $\frac{\infty}{\infty}$, and so we are in a situation where we can use L'Hopitals rule. When we use L'hopitals rule, we get

$$\begin{aligned} \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} &\stackrel{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \\ &= \lim_{x\to\infty} \left(\frac{1}{x}\frac{2\sqrt{x}}{1}\right) \\ &= \lim_{x\to\infty} \frac{2}{\sqrt{x}} \\ &= 0. \end{aligned}$$

Notice that after we use L'Hopitals rule once, we can just do a little simplifying and our limit becomes something we know.

(c) This limit goes to $\frac{0}{0}$, so we can use L'Hopitals rule here. If we use L'Hopitals rule, we get

$$\begin{aligned} \lim_{x\to 1} \frac{x^3 + x^2 - 37x + 35}{x^3 - 2x^2 - 5x + 6} &\stackrel{L'H}{=} \lim_{x\to 1} \frac{3x^2 + 2x - 37}{3x^2 - 4x - 5} \\ &= \frac{3 + 2 - 37}{3 - 4 - 5} \\ &= \frac{-32}{-6} \\ &= \frac{16}{3}. \end{aligned}$$

Notice that after we used L'Hopitals rule once, we got into a situation where we could evaluate the limit. You can't use L'Hopitals rule twice here.