

## Quiz 4, Calc 2, Section 3

February 26, 2007

1. Determine if the following sequences are convergent and/or bounded

(a)  $\langle \cos\left(\frac{n\pi}{2}\right) \rangle$

(b)  $\langle \frac{4^n}{7^n - 5} \rangle$

(c)  $\langle \frac{5n^3 + 15n - 1}{4n^3 - 16n^2} \rangle$

*Solution:* (a) To determine if the sequence converges we need to look at the limit as  $n$  goes to infinity. Now

$$\lim_{n \rightarrow \infty} \cos\left(\frac{n\pi}{2}\right) = DNE$$

since the value of  $\cos(n\pi/2)$  keeps oscillating between the values 1,  $-1$ , and 0. So it never settles on a particular number as  $n$  gets bigger. So the sequence is divergent. Now we need to see if it is bounded. Since  $-1 \leq \cos(x) \leq 1$ , we have

$$-1 \leq \cos\left(\frac{n\pi}{2}\right) \leq 1.$$

So the sequence is bounded. Overall we have that the sequence is divergent and bounded.

(b) Again, the first thing we need to look at is the limit as  $n$  goes to infinity. Here we get

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{4^n}{7^n - 5} &= \lim_{n \rightarrow \infty} \frac{4^n}{7^n \left(1 - \frac{5}{7^n}\right)} \\ &= \lim_{n \rightarrow \infty} \left( \frac{4^n}{7^n} \frac{1}{1 - \frac{5}{7^n}} \right) \\ &= 0 \end{aligned}$$

since  $4^n/7^n$  goes to 0 as  $n$  goes to infinity. So the sequence is convergent to 0, and since it is convergent, it is automatically bounded.

(c) We need to look at the limit as  $n$  goes to infinity, so we have

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{5n^3 + 15n - 1}{4n^3 - 16n^2} &= \lim_{n \rightarrow \infty} \frac{n^3 \left(5 + \frac{15}{n^2} - \frac{1}{n^3}\right)}{n^3 \left(4 - \frac{16}{n}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{5 + \frac{15}{n^2} - \frac{1}{n^3}}{4 - \frac{16}{n}} \\ &= \frac{5 + 0 - 0}{4 - 0} = \frac{5}{4}.\end{aligned}$$

So the sequence converges to  $5/4$ , and since it is convergent it is automatically bounded.

2. Evaluate the following integral

$$\int x \cos(2x) dx.$$

*Solution:* This integral involves two different kinds of functions: a power of  $x$  and a trig function. So we should use integration by parts. We need to choose our  $u$  and  $dv$ . Let  $u = x$  and  $dv = \cos(2x)dx$ . So  $du = dx$  and  $v = \int \cos(2x) dx = \frac{1}{2} \sin(2x)$ . Using the formula

$$\int u dv = uv - \int v du$$

we get

$$\begin{aligned}\int x \cos(2x) dx &= x \frac{1}{2} \sin(2x) - \int \frac{1}{2} \sin(2x) dx \\ &= \frac{x}{2} \sin(2x) - \frac{1}{2} \int \sin(2x) dx \\ &= \frac{x}{2} \sin(2x) - \frac{1}{2} \left( -\frac{1}{2} \cos(2x) \right) + C \\ &= \frac{x}{2} \sin(2x) + \frac{1}{4} \cos(2x) + C.\end{aligned}$$