Quiz 1, Calc 2, Section 9
February 5, 2007

1.Find y' where

(a) $y = \log_2(\sin x)$

(b) $y = e^{\sec(2x)}$

Solution: (a) This is a chain rule situation. The first step which might simplify the problem is to rewrite y as follows,

$$y = \log_2(\sin x) = \frac{\ln(\sin x)}{\ln 2}.$$

Using this rewritten form of y, we get

$$\begin{aligned} y' &= \frac{d}{dx}\left(\frac{\ln(\sin x)}{\ln 2}\right) \\ &= \frac{1}{\ln 2}\frac{1}{\sin x}\frac{d}{dx}(\sin x) \\ &= \frac{\cos x}{\sin(x)\ln 2} \\ &= \frac{\cot x}{\ln 2}. \end{aligned}$$

(b) Again, this is a chain rule situation. The chain rule tells us to take the derivative from the outside in, so we get

$$\begin{aligned} y' &= \frac{d}{dx}\left(e^{\sec(2x)}\right) \\ &= e^{\sec(2x)}\frac{d}{dx}(\sec(2x)) \\ &= e^{\sec(2x)}\sec(2x)\tan(2x)2 \\ &= 2\sec(2x)\tan(2x)e^{\sec(2x)}. \end{aligned}$$

2. Evaluate the following integrals

(a) $\int x^{5x^2}\, dx$.

(b) $\int \cot x\, dx$.

Solution: (a) This integral looks like a prime candidate for a substitution. Let $u = 5x^2$, and so $du = 10x$. Or $(1/10)du = xdx$. Using this substitution, we can rewrite the integral

$$\begin{aligned}\int x3^{5x^2}\ dx &= \frac{1}{10}\int 3^u\ du \\ &= \frac{1}{10}\frac{3^u}{\ln 3} + C \\ &= \frac{1}{10\ln 3}3^{5x^2} + C.\end{aligned}$$

Another way to approach this integral is to write it in terms of e and solving it that way. If we do that, then we get

$$\int x3^{5x^2}\ dx = \int xe^{5x^2\ln 3}\ dx.$$

Here, we can use the substitution $u = 5x^2 \ln 3$, so $du = 10x \ln 3\ dx$. Rewritten we have $(1/10\ln 3)du = x\,dx$. Using this substitution, we get

$$\begin{aligned}\int xe^{5x^2\ln 3}\ dx &= \frac{1}{10\ln 3}\int e^u\ du \\ &= \frac{1}{10\ln 3}e^u + C \\ &= \frac{1}{10\ln 3}e^{5x^2\ln 2} + C.\end{aligned}$$

(b) This probelm becomes a lot easier when we rewrite $\cot x$ in terms of $\sin x$ and $\cos x$.

$$\int \cot s\ dx = \int \frac{\cos x}{\sin x}\,dx.$$

From here, we can use the substitution $u = \sin x$, $du = \cos x\ dx$. This gives

$$\begin{aligned}\int \cot x\ dx &= \int \frac{\cos x}{\sin x}\,dx \\ &= \int \frac{1}{u}\,du \\ &= \ln|u| + C \\ &= \ln|\sin x| + C.\end{aligned}$$