Quiz 3, Calc 2, Section 9

February 26, 2007

1. Determine if the following sequences are convergent and/or bounded

   (a) $\langle \cos\left(\frac{n\pi}{2}\right)\rangle$

   (b) $\langle \frac{4^n}{7^n - 5}\rangle$

   (c) $\langle \frac{5n^3 + 15n - 1}{4n^3 - 16n^2}\rangle$

Solution: (a) To determine if the sequence converges we need to look at the limit as n goes to infinity. Now

$$\lim_{n\to\infty} \cos\left(\frac{n\pi}{2}\right) = DNE$$

since the value of $\cos(n\pi/2)$ keeps oscillating between the values $1, -1$, and 0. So it never settles on a particular number as n gets bigger. So the sequence is divergent. Now we need to see if it is bounded. Since $-1 \le \cos(x) \le 1$, we have

$$-1 \le \cos\left(\frac{n\pi}{2}\right) \le 1.$$

So the sequence is bounded. Overall we have that the sequence is divergent and bounded.

(b) Again, the first thing we need to look at is the limit as n goes to infinity. Here we get

$$\begin{aligned} \lim_{n\to\infty} \frac{4^n}{7^n - 5} &= \lim_{n\to\infty} \frac{4^n}{7^n(1 - \frac{5}{7^n}} \\ &= \lim_{n\to\infty} \left(\frac{4^n}{7^n} \frac{1}{1 - \frac{5}{7^n}}\right) \\ &= 0 \end{aligned}$$

since $4^n/7^n$ goes to 0 as n goes to infinity. So the sequence is convergent to 0, and since it is convergent, it is automatically bounded.

(c) We need to look at the limit as n goes to infinity, so we have

$$\begin{aligned}\lim_{n\to\infty}\frac{5n^3+15n-1}{4n^3-16n^2} &= \lim_{n\to\infty}\frac{n^3\left(5+\frac{15}{n^2}-\frac{1}{n^3}\right)}{n^3\left(4-\frac{16}{n}\right)}\\ &= \lim_{n\to\infty}\frac{5+\frac{15}{n^2}-\frac{1}{n^3}}{4-\frac{16}{n}}\\ &= \frac{5+0-0}{4-0}=\frac{5}{4}.\end{aligned}$$

So the sequence converges to $5/4$, and since it is convergent it is automatically bounded.

2. Evaluate the following integral

$$\int x\cos(2x)\,dx.$$

Solution: This integral involves two different kinds of functions: a power of x and a trig function. So we should use integration by parts. We need to choose our u and dv. Let $u=x$ and $dv=\cos(2x)dx$. So $du=dx$ and $v=\int\cos(2x)\,dx=\frac{1}{2}\sin(2x)$. Using the formula

$$\int u\,dv=uv-\int v\,du$$

we get

$$\begin{aligned}\int x\cos(2x)\,dx = x\frac{1}{2}\sin(2x)-\int\frac{1}{2}\sin(2x)\,dx &\\ &= \frac{x}{2}\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\ &= \frac{x}{2}\sin(2x)-\frac{1}{2}\left(-\frac{1}{2}\cos(2x)\right)+C\\ &= \frac{x}{2}\sin(2x)+\frac{1}{4}\cos(2x)+C.\end{aligned}$$