

Review Sheet for the Final Exam

1. If $f(x) = 3x + \cos x$, find $f^{-1}(1)$ and $(f^{-1})'(1)$.

2. Find a formula for the inverse of the function

$$y = \frac{2x + 3}{4 - 5x}.$$

3. Find $\frac{dy}{dx}$ if

$$y = \frac{x^4 \sqrt[5]{4x^2 - 3}}{(x + 7)^5 \sqrt{x^2 + 1}}.$$

4. Find $f'(x)$ if

$$f(x) = (x^2 - 3)^{x+4}.$$

5. Evaluate the following limits

(a) $\lim_{x \rightarrow 0} (\sin x)^x$

(b) $\lim_{x \rightarrow \infty} x^{(1/x)}$

(c) $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2}$

6. Solve the following equation for x ,

$$\ln(x + 1) - \ln(x - 1) = 1.$$

7. Evaluate the following integrals

(a) $\int x^2 \sin x \, dx$

(b) $\int_0^\pi \sin^2 x \, dx$

(c) $\int \frac{x^3}{\sqrt{4 + x^2}} \, dx$

(d) $\int \sin^5 x \cos^5 x \, dx$

(e) $\int \cot x \ln(\sin x) \, dx$

(f) $\int_5^{13} \frac{1}{(x - 5)^{3/2}} \, dx$

(g) $\int_5^{13} \frac{1}{(x - 5)^{2/3}} \, dx$

(h) $\int_0^\infty \frac{1}{1 + x^2} \, dx$

8. Find the length of the curve $y = \ln(\cos x)$ between $0 \leq x \leq \pi/3$.

9. Find the surface area obtained by rotating the curve $y = \sqrt{x}$ about the x -axis for $4 \leq x \leq 9$.

10. Solve the differential equation

(a) $2(\ln y)y' - xy = 0$

(b) $y' - xe^y = 0$

11. Eliminate the parameter to find a Cartesian equation of the curve

(a) $x = t^2 - 2, y = 5 - 2t$

(b) $x = t^2, y = t^3$

12. Find dy/dx and d^2y/dx^2 for the two curves in the previous problem. What is the equation of the tangent line to each curve at $t = 1$?

13. Determine if the following series converge or diverge and justify your answer

(a) $\sum_{n=0}^{\infty} (\cos 1)^n$

(b) $\sum_{n=3}^{\infty} \frac{\sqrt{n^2 + 4}}{4n^3 - 6n}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\sum_{n=1}^{\infty} \left(\frac{3 - 4n^3}{5n^2 + 1} \right)^n$

(e) $\sum_{n=3}^{\infty} \frac{2^n - 3}{5^n + n}$

14. Determine if the following series converge absolutely, converge conditionally, or diverge and justify your answer.

(a) $\sum_{n=2}^{\infty} \frac{\sin n}{n^2}$

(b) $\sum_{n=5}^{\infty} (-1)^n \frac{\ln n}{n^3}$

(c) $\sum_{n=1}^{\infty} \frac{n+2}{3^n}$

(d) $\sum_{n=3}^{\infty} \frac{(-1)^n}{\sqrt{n+5}}$

(e) $\sum_{n=13}^{\infty} \frac{5^n n^3}{n!}$

15. Find the radius and interval of convergence for the following power series

(a) $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n4^n}$

(b) $\sum_{n=0}^{\infty} \frac{x^n}{4^n \ln n}$

(c) $\sum_{n=0}^{\infty} (-1)^n n 4^n x^n$

(d) $\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n^2 5^n}$

16. Find the degree 4 Taylor polynomial for $f(x) = \ln(x+4)$ around $a = 2$.

17. Find the degree 3 Taylor polynomial for $f(x) = \cos x$ around $a = \pi/4$.

18. Find the Maclaurin series for $f(x) = 3x^3 - 4x + 2$.