

Homework 4, Topology

Due Monday October 22nd, 2007

Justify your answers.

1. Let A and B be subsets of a topological space X . Prove the following

(i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(ii) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$

(iii) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$

(iv) $A^\circ \cap B^\circ = (A \cap B)^\circ$

Give counterexamples to equality in (ii) and (iii).

2. Let X be a metric space with metric d . Show that $d : X \times X \rightarrow \mathbb{R}$ is continuous. Here \mathbb{R} is the real numbers with the standard topology.

3. Let X be a set, and define a function $d : X \times X \rightarrow \mathbb{R}$ by

$$d(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

(i) Show that d is a metric.

(ii) What topology does d induce on X ?

4. Prove that if X is not Hausdorff then there is a pair of points $x, y \in X$ and a sequence which converges to x and also converges to y .

5. Show that \mathbb{R} with the finite complement topology isn't Hausdorff.

6. Let A be a subset of a topological space X .

(i) Prove that

$$\overline{A} \cap \overline{(X - A)} = \overline{A} - A^\circ.$$

This shows that the two descriptions we gave in class for the boundary of A are equivalent.

(ii) If A is an open set, is it true that $A = \text{Int}(\overline{A})$?