

Homework 5, Topology

Due Friday November 16, 2007

Justify your answers.

1. Show that the torus T is path connected.
2. Let \mathcal{T} and \mathcal{T}' be two topologies on X with $\mathcal{T} \subset \mathcal{T}'$.
 - (a) What does connectedness of X in one topology imply about connectedness of X in the other topology?
 - (b) What does compactness of X in one topology imply about compactness of X in the other topology?
3. Let $p : X \rightarrow Y$ be a quotient map. Show that if each set $p^{-1}(\{y\})$ is connected, and Y is connected, then X is connected.
4. Let \mathbb{R}_{fc} denote the real numbers with the finite complement topology, and let \mathbb{R}_ℓ denote the real numbers with the half-open (lower limit topology).
 - (a) Is \mathbb{R}_{fc} connected? Is \mathbb{R}_{fc} path connected?
 - (b) Is \mathbb{R}_ℓ connected? Is \mathbb{R}_ℓ path connected?
5. Let X and Y be topological spaces, X be path connected, and $f : X \rightarrow Y$ be a continuous map. Is $f(X)$ necessarily path connected?
6. Show that if U is an open connected subspace of \mathbb{R}^2 , then U is path connected. [Hint: Given a point $x_0 \in U$, show that the set of points that can be joined to x_0 by a path in U is both open and closed in U .]