

This review sheet is not complete. There are things that we covered that can be on the final which are not on this review sheet. Any topic covered during the semester is fair game. Use the homework, previous exams, previous practice exams with solutions, and all the supplemental material on the web for more help in studying.

- Read **all** problems first. Try working them from easiest to hardest.
 - In order to get any credit, you must **show all your work**. Explain your answers.
1. For the case of “One Man, One Vote” we talked about four voting methods and four fairness criteria.
 - (a) List and describe the four voting methods.
 - (b) List and define the four fairness criteria.
 - (c) Say which methods satisfy which criteria.
 - (d) Which voting method satisfies all four fairness criteria? Explain your answer.
 2. The preference schedule for an election with four candidates and 28 voters is given in the table below.

Choice	Number of ballots			
	9	8	6	5
1st	A	C	D	B
2nd	B	D	B	D
3rd	C	B	A	C
4th	D	A	C	A

- (a) Determine the winner using the **Borda vote count** method.
- (b) Determine the winner using the **plurality with elimination** method.
- (c) Determine the winner using the **pairwise comparison** method.
- (d) Rank the candidates from first to fourth using the **plurality with elimination** method.
3. In a weighted voting system, what is a dummy voter? What is a dictator?
4. Consider the weighted voting system $[25 : 10, 9, 8, 5, 2]$ with voters $A, B, C, D,$ and E .
 - (a) How many voters are there? How many coalitions are there?
 - (b) List all the winning coalitions and their weight.
 - (c) Underline the critical voters in each winning coalition.
 - (d) In how many winning coalitions is C a critical voter?
 - (e) Are there any dummy voters?
 - (f) Does anyone have veto power?
 - (g) Find the power index of each voter in this system.
5. The Republic of Novindus is made up of five autonomous Cities: Aen, Baid, Cylt, Denyr, and Eta. The republic has a legislature with 85 seats. The seats are to be apportioned according to the population of the cities. The table below gives the population of the cities in millions of people.

City	Aen	Baid	Cylt	Danyr	Eta
Population	12.78	8.25	11.32	6.67	7.82

- (a) Find the **standard divisor**.
- (b) Find the apportionment using **Hamilton’s method**.

- (c) Find the apportionment using **Jefferson's method**.
- (d) Find the apportionment using **Adam's method**.
- (e) Find the apportionment using **Webster's method**.
- (f) Find the apportionment using the **Huntington-Hill method**.
6. Describe the different apportionment methods we talked about. What are the differences? Were any used by Congress?
7. What is the Alabama Paradox? What is the quota criterion?
8. What is a fair division? What is an envy-free fair division?
9. Sam, Fred, and Betty buy a pizza for \$36. It is half pepper and half onion. Betty likes onion twice as much as she likes pepper, while Fred likes pepper and onion the same. They decide to divide up the pizza using the cut and choose method. In the first round, Sam will cut and Fred will choose with Betty joining in for round 2. Sam cuts the pizza into two pieces. The first piece consists of $\frac{2}{5}$ of the onion half and 70% of the pepper half.
- (a) Which topping does Sam prefer?
- (b) Which of the two pieces will Fred choose? Justify your answer by showing what Fred thinks each of the two pieces is worth.
- (c) Show one possible way that Fred might divide up his piece for Betty in round 2.
- (d) In the division you made in part (b), show what Betty thinks each piece is worth. Which piece will Betty choose?
10. Mark and Bruce are dividing up a plot of land, but due to the how much each invested Bruce has rights to 80% of the land, and Mark only has rights to 20% of the land. Describe in detail how they can divide the land using a variation of the cut and choose method.
11. Four siblings, Adam, Joe, Laura, and Zach started an antique store together. Due to bad sales, they decide to divide up the business. The business has assets of a store, the stock of antiques, a warehouse, and a truck. They decide to use the sealed bids method to divide up the assets. Their bids on the items are given in the table below.

	Adam	Joe	Laura	Chuck
Store	250,000	252,000	263,000	257,000
Antiques	790,000	722,000	728,000	770,000
Warehouse	150,000	162,000	157,000	174,000
Truck	35,000	35,300	37,700	36,500

Assuming that they have equal rights to the business, answer the following questions.

- Who gets each item?
- How much is a fair share for each person?
- How much cash does each of them pay, or get paid?
- How much is the total surplus?
- What is the final allocation to each one? What items and how much cash?

Assuming Adam owns 40% of the business, Joe owns 30%, Laura owns 15% and Zach owns 15% of the business, answer the following questions.

- Who gets each item?
- How much is a fair share for each person?
- How much cash does each of them pay, or get paid?
- How much is the total surplus?

- What is the final allocation to each one? What items and how much cash?
12. William is at a deli that serves sandwiches. The deli has four kinds of bread, fifteen kinds of meat, five vegetables (tomato, lettuce, onion, cucumber, and pepper), and four optional toppings (mayonnaise, mustard, hot sauce, and cheese). A ‘sandwich’ consists of a choice of bread, a choice of meat, a choice of two vegetables, and the toppings are optional.
 - (a) How many different possible sandwiches can William order?
 - (b) A ‘carnivore sandwich’ consists of a choice of bread, four different kinds of meat, and all vegetables are optional and all toppings are optional. How many ‘carnivore sandwiches’ are possible?
 - (c) A ‘veggie sandwich’ consists of a choice of bread, at least one vegetable (so could have more than one vegetable) and all of the toppings are optional. How many ‘veggie sandwiches’ are possible?
 13. Two different dice are rolled and the numbers on top are observed. Consider the event **E**: “the product of the two numbers is a multiple of 3.”
 - (a) Write **E** as a set of outcomes.
 - (b) If the two dice are fair, what is the probability of **E**?
 - (c) If both die have the probability space

Outcome	1	2	3	4	5	6
Probability	0.2	0.15	0.1	0.35	0.1	0.1

What is the probability of **E**?

14. A deck of cards comes with 52 cards in 4 suits (Hearts, Clubs, Spades, and Diamonds). Each suit has 13 values from 2, which is lowest, to Ace, which is highest. A hand of 6 cards is drawn from a well-shuffled deck of 52 cards.
 - (a) How many different hands are possible?
 - (b) How many possible hands contain four cards of one value and two cards of another value?
 - (c) How many possible hands contain six consecutive cards from one suit?
 - (d) What is the probability of getting three pairs in the hand of six cards? A pair is two cards with the same value in different suits.
 - (e) What is the probability of getting six consecutive cards from one suit?
15. A collection of 45 index cards are labeled with the numbers 1 to 45. They are shuffled and 6 cards are dealt. What is the probability that the value on each of the cards is divisible by 3?
16. John is takes a multiple choice quiz, which he hasn’t studied for, and he guesses on every question. There are 12 questions, and each question has 5 choices. What is the probability that John gets 2 questions right and 10 wrong?
17. Explain the meaning of each of the following terms:
 - (a) Data set
 - (b) Data point
 - (c) Coordinate
 - (d) Variable
 - (e) Quantitative variable
 - (f) Qualitative variable
 - (g) Median

- (h) Mean
- (i) Variance
- (j) Standard Deviation
- (k) Normal Curve
- (l) Non-normal Distribution

18. The raw data from an Intro to Phrenology exam is given below:

25,	30,	37,	22,	17,	29,	36,	30,	27,	36,
26,	28,	19,	44,	27,	28,	32,	32,	22,	36,
28,	20,	33,	29,	30,	26,	21,	35,	28,	32,
31,	29,	28,	32,	23,	27,	37,	21,	30,	23,
29,	27,	23,	36,	31,	29,	28,	31,	27,	21,
34,	28,	79,	31,	27,	20,	32,	29,	34,	29,

- (a) Sort the data in ascending order.
 - (b) Make a frequency table for the data set. Be sure to include a row for cumulative frequency.
 - (c) Draw the bar graph diagram for this data set.
19. Using the frequency table that you made in the previous problem
- (a) Find the median, the lower half and the upper half.
 - (b) Find the first and third quartiles.
 - (c) Determine any outliers.
 - (d) Draw the boxplot with outliers.
20. Using the data set in problem 18, group the scores in class intervals of length 10 and draw the corresponding bar graph diagram.
21. Using the data set in problem 18, group the scores using the class intervals $[0, 13)$, $[13, 25)$, $[25, 33)$, $[33, 45)$, and $[45, 80]$ and draw the corresponding histogram.
22. Find the mean and standard deviation of the data set in problem 18.
23. The Clearview Hospital has determined that the ages of all of the citizens of Clearview follow a near-normal distribution with $\mu = 43$ years, and $\sigma = 12.5$ years.
- (a) If a citizen of Clearview is randomly chosen, what is the probability that they are over 68 years old?
 - (b) What can you say about a 30.5 year old citizen?
 - (c) Estimate the lower and upper quartiles for the citizens' ages.
24. You have a large jar of gumballs. You want a rough estimate of how many gumballs you have, but you don't want to count all of the gumballs, so you decide to use the capture-recapture method to get an estimate.
- For the first sample you take out 180 gumballs and mark them with a scratch. The 180 gumballs are 75 red, 55 blue, and 50 green. You put the gumballs back in the jar and shake it well.
- A second sample of 220 gumballs is randomly chosen. It consists of 84 red, of which 24 are marked; 79 blue, of which 15 are marked; and 57 green, of which 12 are marked.
- (a) Estimate the number of red gumballs in the jar.
 - (b) Estimate the number of blue gumballs in the jar.
 - (c) Estimate the number of green gumballs in the jar.
 - (d) Estimate the total number of gumballs in the jar by adding together your answers to parts

(*a*), (*b*), and (*c*).

(e) Estimate the total number of gumballs in the jar by using the capture-recapture method on the total number of gumballs.

25. A coin is tossed 1,000 times and it comes up heads 650 times. Can we safely conclude that the coin is not fair? With what confidence level?