Exam 1 Calc III, Section 01

February 24, 2009

Rules for the Exam:

- No calculators are permitted for the exam.
- No books or notes can be used during the exam.
- Clearly mark your final answer for every problem.
- Show all of your work.

1. Consider the following two planes:

$$x + 4y - 2z = 12$$

$$2x + 3y + 2z + 18 = 0$$

   (a) (5 points) Find the exact value of the angle between the two planes.

   (b) (15 points) Find the parametric equations for the line of intersection of the planes.

2. (10 points) Consider the points $P(1, 1, 5)$, $Q(2, 4, -2)$ and $R(3, 5, 2)$. Find the linear equation of the plane containing P, Q, and R.

3. (10 points) Show that the lines with parametric equations

$$\begin{array}{c} x = 2 - t \\ y = 3t + 1 \\ z = 6 - 8t \end{array} \quad \text{and} \quad \begin{array}{c} x = -1 + s \\ y = 2 + s \\ z = -8 + 3s \end{array}$$

   intersect in a single point and find the point.

4. Consider the following space curve

$$\mathbf{r}(t) =< t^2 - 4, \qquad t2\sqrt{3} + 7, \qquad \frac{4\sqrt{2}}{3}(t-1)^{3/2} > .$$

   (a) (10 points) Find the arc length function of the curve measured from when $t = 0$ in the direction of increasing t.

   (b) (5 points) Reparametrize the curve with respect to arclength using your previous answer.

5. Consider the following space curve

$$\mathbf{r}(t) =< \sin(2t), \qquad 3t - 4, \qquad \cos(2t) > .$$

   (a) (5 points) Find the unit tangent vector $\mathbf{T}(t)$.

   (b) (10 points) Find the curvature of $\mathbf{r}(t)$.

6. Find the limit if it exists or show that the limit doesn't exist.

   (a) (10 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3 + y^3}$.

   (b) (5 points) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2 + xy - 2y^2}{x^2 - y^2}$.

   (c) (5 points) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy + y^2 - 2}{x^2 - y^2}$.

7. (10 points) Evaluate the following

$$\int_0^5 \left(e^{2t}\,\mathbf{i} - \frac{4}{9-t}\,\mathbf{j} + \sqrt{5t}\,\mathbf{k} \right) dt.$$