

## Exam 2

March 24, 2009

## Calc III, Section 01

Rules for the Exam:

- No calculators are permitted for the exam.
- No books or notes can be used during the exam.
- Clearly mark your final answer for every problem.
- Show all of your work.

1. (a) (5 points) Evaluate the following integral

$$\iint_R e^{x+y} dA$$

where  $R = \{(x, y) \mid 0 \leq x \leq 3, 2 \leq y \leq 5\}$ .

- (b) (15 points) Evaluate the following integral

$$\iint_D (2x + 1) \sin(x + y) dA$$

where  $D$  is the region of the  $xy$ -plane bounded by  $y = x^2$ , the line  $x = \pi$ , and the  $x$ -axis.

2. (10 points) Sketch the domain of integration and change the order of integration in the following integral

$$\int_1^2 \int_0^{\ln x} f(x, y) dy dx.$$

3. Let  $f(x, y) = e^{y^2} \cos(\pi x) - \sin(xy)$  where

$$x(s, t) = \ln(st), \quad \text{and} \quad y(s, t) = \frac{s - st}{s + t}$$

- (a) (10 points) Find  $\frac{\partial f}{\partial t} \Big|_{s=1, t=1}$ .

- (b) (10 points) Find  $\frac{\partial f}{\partial s}$ . Your answer can be in terms of  $x, y, s$  and  $t$ .

4. (10 points) Let  $F(x, y, z) = 3xy - z^2 + 4xy^2 - y^2$  and  $\vec{v} = \langle 1, 1, -4 \rangle$ . Find the directional derivative of  $F$  in the direction of  $\vec{v}$  at the point  $(2, 1, 3)$ .

5. (10 points) Find the equation for the tangent plane to the surface

$$3xy^2 - 8yz + 4xz^2 + y^2 = 4$$

at the point  $P(1, 1, 2)$ .

6. (15 points) Find all the relative maxima, minima and saddle points of the function

$$f(x, y) = x^2y - 6y^2 - 3x^2.$$

7. (15 points) Let  $f(x, y) = x^2 + y^2$ . Find the absolute maximum and absolute minimum values of  $f(x, y)$  on the domain

$$D = \{(x, y) \mid x^2 + xy + y^2 \leq 3\}.$$