Review Sheet for Exam 3, Calc III

This is a list of some practice problems for the third exam. I don't claim that this is a complete list covering all possible types of problems, but it is a start. You should also use the textbook as a resource for practice problems.

1. Find the volume of the solid enclosed by the surface $z = (x-3)^2 + (y-2)^2$ and the plane $z = 4$.
2. Find the volume of the solid enclosed by the surface $z = x^2 \sec^2(y)$ and the planes $z = 0$, $x = 2$, $x = 4$, $y = \pi/4$ and $y = \pi/3$.
3. Find the volume of the solid bounded by the parabaloid $z = x^2 + y^2$ and the hemisphere $z = \sqrt{6 - x^2 - y^2}$.[Hint: The intersection of the two surfaces lies on the plane $z = 2$.]
4. Evaluate the double integral
   (a) $\int\int_D xy\, dA$ where D is bounded by $y = \sqrt{x+1} - 1$ and $y = (x+1)^3 - 1$.
   (b) $\int\int_D y\sin(x)\, dA$ where D is bounded by the curves $y^2 - x - 4 = 0$ and $x - y + 2 = 0$.
5. Find the volume of the solid under the surface $z = x + y$ and above the region bounded by the curves $y = x^2 + x$ and $y = x^3 - x$ for $x \geq 0$. [Hint: $x^2 + x = x^3 - x$ when $x = 2$.]
6. Evaluate the double integral
$$\int\int_D e^{-x^2-y^2}\, dA$$
   where D is the region to the left of the y-axis bounded by the curves $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$.
7. Evaluate the double integral
$$\int_0^1 \int_{\arcsin(y)}^{\pi/2} \cos(x)\sqrt{1 + \cos^2(x)}\; dxdy$$
   by switching the order of integration.
8. Evaluate the triple integral
   (a) $\int\int\int_E xy\, dV$ where E is the solid tetrahedron with verticies $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.
   (b) $\int\int\int_E 2z\, dV$ where E is the solid region bounded by the cylinder $z = x^2$ and the planes $y = 0$ and $z + y = 1$.
   (c) $\int\int\int_E x + z\, dV$ where E is the solid region in the first octant bounded by the cylinder $X^2 + y^2 = 1$ and the planes $y = z$, $x = 0$, and $z = 0$.
   (d) $\int\int\int_E xy\, dV$ where E is the solid region bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.
9. Find the volume of the region above the parabaloid $z = x^2 + y^2$ and below the parabaloid $z = 36 - 3x^2 - 3y^2$.
10. Evaluate the integral
$$\int\int\int_E (x^2 + y^2)z\, dV$$
    where E is the region above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 2z$.
11. Evaluate the integral
$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\, dz\, dy\, dx$$
    by changing to cylindrical coordinates.
12. Find the Jacobian of the transformation $x = e^u \cos v$ and $y = e^u \sin v$.
13. Find the Jacobian for the transformation $x = u + v + w$, $y = u + v - w$, and $z = u - v + w$.
14. Find the image of the set S under the transformation $x = 4u + 3v$ and $y = 4v$ where S is the triangular region on the uv-plane with verticies $(0,0)$, $(1,0)$, and $(0,1)$.

15. Let R be the region of the xy-plane bounded by the lines $y = x$, $y = x - 2$, $y = -2x$, and $y = 3 - 2x$. Evaluate

$$\int\int_R (x + 4y)\, dA$$

by using the transformation

$$x = \frac{1}{3}(u + v) \quad \text{and} \quad y = \frac{1}{3}(v - 2u).$$

16. Evaluate the integral

$$\int\int_R xy\, dA$$

where R is the region in the first quadrant bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$ and $xy = 3$. Use the transformation given by $u = xy$ and $v = y/x$.

17. Evaluate the integral

$$\int\int_R \cos\left(\frac{y - x}{y + x}\right)\, dA$$

where R is the trapazoidal region with verticies $(1, 0)$, $(0, 1)$, $(0, 2)$, and $(2, 0)$. Try using an obvious transformation.

18. Sketch the vector field $\vec{\mathbf{F}}(x, y) =< y, 1 >$ on the region $[0, 2] \times [0, 2]$.

19. Find the gradient vector field for the following functions

    (a)
    $$f(x, y) = e^x y \sin(x/y).$$

    (b)
    $$f(x, y, z) = 4x^2y^3z - 6xyz^2 + \ln(xyz).$$

    (c)
    $$f(x, y) = \sin(e^{xyz}) + 15x^2y^2 - z.$$

20. Evaluate the integral

    (a) $\int_C y\, ds$, where C is given by the parametrization $x = t^2$, and $y = t$ for $0 \leq t \leq 1$.

    (b) $\int_C xy^2z\, ds$, where C is the line segment connecting the points $(1, 0, 1)$ and $(0, 3, 6)$.

    (c) Evaluate the line integral

    $$\int_C x^3y^2z\, dz$$

    where C is given by $x = 2t$, $y = t^2$, and $z = t^2$ for $0 \leq t \leq 1$.

    (d) Evaluate the line integral

    $$\int_C y^2\, dx + x^2\, dy$$

    where the curve C is the upper half of the circle $x^2 + y^2 = 4$ combined with the line segment between $(-2, 0)$ and $(2, 4)$.

21. If $\vec{\mathbf{F}}(x, y) = xy\vec{\mathbf{i}} + 6\cos(y)\vec{\mathbf{j}}$, evaluate

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

where C is given by $\vec{\mathbf{r}}(t) =< 3t, 4t^2 >$ for $0 \leq t \leq \sqrt{\pi}$.

22. Determine if the vector field

$$\vec{\mathbf{F}}(x, y) =< 1 + 4x^3y^3, 3x^4y^2 - 2y >$$

is conservative. If it is conservative, find the potential function f with $\vec{\mathbf{F}} = \vec{\nabla} f$.

23. (a) Find a function f such that $\vec{\mathbf{F}} = \vec{\nabla} f$ for $\vec{\mathbf{F}} =< y, x + z, y >$.

    (b) Use (a) to evaluate

    $$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

    where C is given by $\vec{\mathbf{r}}(t) =< \ln(5t^2 + 1)\sin(t\pi), e^2t + t^2, 3t^2 - 4t + 1 >$ for $0 \leq t \leq 2$.

24. (a) Find a function f such that $\vec{\mathbf{F}} = \vec{\nabla} f$ for $\vec{\mathbf{F}} =< x^2y^3, x^3y^2 >$.
    (b) Use (a) to evaluate

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

where C is given by $\vec{\mathbf{r}}(t) =< \sin(t\pi/4), 4t^3 - 8t + 2^t >$ for $0 \le t \le 2$.

25. Show that the line integral

$$\int_C 2x\sin(y)\,dx + (x^2\cos(y) - 3y^2)\,dy$$

where C is any path from $(-1, 0)$ to $(5, 1)$ is independent of the path and evaluate the integral.

26. Evaluate the line integral

$$\oint_C (x + 2y)\,dx + (x - 2y)\,dy$$

where C consists of the arc of the parabola $y = x^2$ where $0 \le y \le 1$ followed by the line segment form $(1, 1)$ to $(0, 0)$ by two methods

    (a) Directly
    (b) Using Green's Theorem

27. Use Green's Theorem to evaluate

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}.$$

Check the orientation of the curve before applying the theorem.

    (a) $\vec{\mathbf{F}}(x, y) =< e^x + x^2y, e^y - xy^2 >$ where C is the circle $x^2 + y^2 = 25$ oriented clockwise.
    (b) $\vec{\mathbf{F}}(x, y) =< y + e^{\sqrt{x}} - \sin(\ln(x^2 + 4)), 2x + 4^{y^2}\cos(y^2) >$ where C is the boundary of the region enclosed by the parabolas $x = y^2$ and $y = x^2$ and C is positively oriented.
    (c) $\vec{\mathbf{F}}(x, y) =< y - \ln(x^2 + y^2), 2\arctan(y/x) >$ where C is the circle $(x - 2)^2 + (y - 3)^2 = 1$ oriented counterclockwise.

28. Use Green's Theorem to evaluate

$$\int_{C_1} xe^{-2x}\,dx + (x^4 + 2x^2y^2)\,dy + \int_{C_2} xe^{-2x}\,dx + (x^4 + 2x^2y^2)\,dy$$

where C_1 is the circle $x^2 + y^2 = 4$ oriented counterclockwise, and C_2 is the circle $x^2 + y^2 = 1$ oriented clockwise.