

### Exam 3

April 28, 2009

### Calc III, Section 01

Rules for the Exam:

- No calculators are permitted for the exam.
- No books or notes can be used during the exam.
- Clearly mark your final answer for every problem.
- Show all of your work.

1. (10 points) Evaluate the line integral

$$\int_C x^{3/2} e^y ds$$

where  $C$  is the piece of the curve  $x = e^{2y}$  going from  $y = 3$  to  $y = 5$ .

2. (10 points) Let  $R$  be the region bounded by  $xy = 1$ ,  $xy = 2$ ,  $xy^2 = 2$ , and  $xy^2 = 4$ . Use the substitution  $x = \frac{u^2}{v}$ ,  $y = \frac{v}{u}$ . **Set up but do not solve** an integral to compute

$$\iint_R (xy^2 - xy) dA$$

using this transformation.

3. Determine if the following vector fields are conservative. If they are, find their potential functions.

(a) (10 points)  $\vec{F}(x, y) = \langle e^x \cos(y), e^x \sin(y) \rangle$ .

(b) (10 points)  $\vec{F}(x, y) = \langle 12xy + y \sin(x), 6x^2 + 30y - \cos(x) \rangle$ .

4. (15 points) Evaluate the integral

$$\iiint_E (xyz) dV$$

where  $E$  is the solid region bounded by the cylinder  $x^2 + z^2 = 4$ , the plane  $y = 2$ , the plane  $y = z$ , and the plane  $z = 0$ .

5. (10 points) Find the volume under the plane  $z = y$  and above the region of the  $xy$ -plane in the first quadrant bounded by the curve  $x^2 + y^2 = 6$ , the curve  $y = x^2$ , and the  $y$ -axis.

6. (15 points) Let  $\vec{F}(x, y) = \langle \frac{2x}{x^2 + e^y}, \frac{e^y}{x^2 + e^y} \rangle$ . Compute the integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where  $C$  is the curve given by

$$\vec{r}(t) = \langle \ln(3t^2 + 1) \sin\left(\frac{t\pi}{2}\right), e^{t^2 - 6t + 8} \cos\left(\frac{t\pi}{2}\right) \rangle$$

for  $2 \leq t \leq 4$ .

7. (10 points) Use spherical coordinates to evaluate the integral

$$\iiint_E z^2 dV$$

where  $E$  is the solid hemisphere above the plane  $z = 0$  bounded by  $x^2 + y^2 + z^2 = 1$ .

8. (10 points) Use Green's Theorem to evaluate the line integral

$$\int_C (3y - e^{\sin(x)}) dx + (7x + \sqrt{y^4 + 1}) dy$$

where  $C$  is the curve given by the piece of  $y = x^3$  from  $x = 0$  to  $x = 1$  followed by the line segment from  $(1, 1)$  to  $(0, 0)$ .