Homework 3, Calc III

Due Feb 18th, 2009

1. Sketch the space curve given by the vector equation

$$\vec{r}(t) = t\vec{i} + \cos(t)\vec{j} + \sin(t)\vec{k}.$$

   Indicate the direction of increasing t on the curve.

2. For the following parametrized curves find $\vec{r'}(t)$ and $\vec{T}(t)$:

   (a)
$$\vec{r}(t) =< t^3, t^2 + 2t - 5, \sin(4t) > .$$

   (b)
$$\vec{r}(t) =< e^t \cos(t), e^t \sin(t), \ln(t^2) > .$$

3. Find the length of the following curve from the point corresponding to $t = 0$ to the point corresponding to $t = \pi/2$.
$$\vec{r}(t) =< 2\cos(t), e, 2t - 2\sin(t) > .$$

4. Find the curvature $\kappa(t)$ for the following space curves:

   (a)
$$\vec{r}(t) = t^2\vec{i} + e^{2t}\vec{j} + \cos(6t)\vec{k}.$$

   (b)
$$\vec{r}(t) =< \cos(2t), 3\sin(2t), 5t > .$$

5. Find the parametric equations for the tangent line to the space curve

$$\vec{r}(t) =< \sin(t), 2e^t + t, \cos(5t) >$$

   at the point $(0, 2, 1)$.

6. For the plane curve
$$\vec{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\vec{i} + \frac{2t}{t^2+1}\vec{j}$$

   (a) Find the function for the arclength of the curve measured from the point $(1, 0)$ in the direction of increasing t.

   (b) Use part (a) to reparametrize the curve with respect to arclength measured from the point $(1, 0)$ in the direction of increasing t. Express the reparametrization in its simplest form.

   (c) Extra Credit: Sketch the curve on the xy-plane with arrows in the direction of increasing t.