Homework 5, Calc III

Due March 11th, 2009

1. Use the chain rule to find dz/dt.
   (a) $z = \arctan(y/x)$, $x = e^t$, $y = 1 - e^{-t}$.
   (b) $z = \dfrac{x^2 - 2xy - y^2}{x^2 + y^2}$, $x = sin(2t^2)$, $y = \cos(2t^2)$.
2. Use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$.
   (a) $z = e^x \sin(y) + \ln(xy)$, $x = \sqrt{s^2 + t^2}$, $y = st + s + t$.
   (b) $z = \sin(u/v) + \cos(v/u)$, $u = e^{2s} + e^{2t}$, $v = \arctan(s + t)$.
3. Consider the function
$$f(x, y) = e^{xy}(x + y).$$
   (a) Find the gradient $\vec{\nabla} f$ of f.
   (b) Find the directional derivative of f in the direction of $\vec{v} =< 2, -1 >$ at the point $(2, 2)$.
   (c) Find the maximum rate of change of f at the point $(1, 3)$ and the direction in which it occurs.
4. Consider the function
$$F(x, y, z) = \sqrt{xyz}$$
   and the level surface of that function given by the equation $F(x, y, z) = 6$.
   (a) Find the equation of the tangent plane to the level surface at the point $(2, 6, 3)$.
   (b) Give the parametric equations for the normal line to the level surface at the point $(3, 2, 6)$.
5. Show that $f(x, y) = 18x^2 - 12xy + 2y^2 + 5$ has an infinite number of critical points. Can you say if the critical points are local maxima or minima? Explain.
6. Find all the local maximum and minimum values and saddle points of the following functions.
   (a) $f(x, y) = xy + \dfrac{1}{x} + \dfrac{1}{y}$.
   (b) $f(x, y) = \dfrac{xy}{x + y}$
   (c) $f(x, y) = e^y(y^2 - x^2)$