Homework 8, Calc III

Due Friday May 8th, 2009

1. Find the curl and divergence of the following vector fields

   (a) $\vec{\mathbf{F}}(x, y, z) = e^{xy}\sin(z)\vec{\mathbf{j}} + y\tan^{-1}(x/z)\vec{\mathbf{k}}$.

   (b) $\vec{\mathbf{F}}(x, y, z) = < e^{x}, e^{xy}, e^{xyz} >$.

2. Determine if the following vector field is conservative. If it is conservative, then find the potential function

$$\vec{\mathbf{F}}(x, y, z) = e^{z}\vec{\mathbf{i}} + ze^{y}\vec{\mathbf{j}} + (xe^{z} + e^{y})\vec{\mathbf{k}}.$$

3. Find the area of helicoid with the vector equation

$$\vec{\mathbf{r}}(u, v) = u\cos(v)\vec{\mathbf{i}} + u\sin(v)\vec{\mathbf{j}} + v\vec{\mathbf{k}}$$

   for $0 \leq u \leq 1$, and $0 \leq v \leq \pi$.

4. Evaluate the surface integral

$$\int\int_S yz\, dS$$

   where S is the part of the plane $x + y + z = 1$ that lies in the first octant.

5. Use Stokes Theorem to evaluate

$$\int\int_S \text{curl}\vec{\mathbf{F}} \cdot dS$$

   where

$$\vec{\mathbf{F}}(x, y, z) = x^2y^3z\,\vec{\mathbf{i}} + \sin(xyz)\,\vec{\mathbf{j}} + xyz\,\vec{\mathbf{k}}$$

   and S is the part of the cone $y^2 = x^2 + z^2$ between the planes $y = 0$ and $y = 3$ oriented in the direction of the positive y-axis.