# Exam 1, Number Systems
September 26th, 2008

Axioms for the integers:
**Axiom 1**. For all integers $m$, $n$, and $p$:

(i) $m + n = n + m$.

(ii) $(m + n) + p = m + (n + p)$.

(iii) $m(n + p) = mn + mp$.

(iv) $mn = nm$.

(v) $(mn)p = m(np)$.

**Axiom 2** There is an integer $0 \in \mathbb{Z}$ such that for all $m \in \mathbb{Z}$, $m + 0 = m$.

**Axiom 3** There is an integer $1 \in \mathbb{Z}$ such that $1 \neq 0$ and for all $m \in \mathbb{Z}$, $m1 = m$.

**Axiom 4** For each $m \in \mathbb{Z}$ there exists an integer, denoted $-m$, such that $m + (-m) = 0$.

**Axiom 5** For all integers $m$, $n$, and $p$, if $m \neq 0$ and $mn = mp$, then $n = p$.

Axiom for the natural numbers:
**Axiom 1** There is a subset $\mathbb{N} \subseteq \mathbb{Z}$ with the following properties:

(i) $1 \in \mathbb{N}$.

(ii) If $n \in \mathbb{N}$, then $n + 1 \in \mathbb{N}$.

(iii) $0 \notin \mathbb{N}$.

(iv) For every $n \in \mathbb{Z}$ such that $n \neq 0$, we have $n \in \mathbb{N}$ or $-n \in \mathbb{N}$.

(v) If a subset $A \subseteq \mathbb{Z}$ satisfies $(i)$ and $(ii)$, then $\mathbb{Z} \subseteq A$.

**PROBLEMS**

1. Prove the following: If $m, x_1, x_2 \in \mathbb{Z}$ satisfy the equations $m + x_1 = 0$ and $m + x_2 = 0$, then $x_1 = x_2$.
2. Prove the following: For all integers $m$ and $n$, $(-m)(-n) = mn$.
3. What is the contrapositive of the statement "If I am a little teapot, then I am short and stout"?
4. For two integers $n$ and $m$, what does it mean to say that $n$ is divisible by $m$?
5. Prove that for all $n \in \mathbb{N}$, $n^3 - n$ is divisible by 3.
6. How did we define the notation $m < n$ for two integers $m$ and $n$?
7. For two sets $A$ and $B$, what does $A \subseteq B$ mean?