

Exam 2, Number Systems

November 14th, 2008

Here are some definitions.

Let $f : A \rightarrow B$ be a function between sets, and let $C \subseteq A$ be a subset of A and $D \subseteq B$ be a subset of B . We define the image of C under f by

$$f(C) = \{b \in B \mid \text{there exists } a \in C \text{ with } f(a) = b\}.$$

We define the preimage of D under f to be

$$f^{-1}(D) = \{a \in A \mid f(a) \in D\}.$$

PROBLEMS

1. What is the definition of a surjective function? What is the definition of an injective function?
2. Let $n \in \mathbb{N}$ and $x, y \in \mathbb{Z}$. Give the definition of $x \equiv y \pmod{n}$ and show that it defines an equivalence relation on \mathbb{Z} .
3. I will define a relation on all the words in the english language as follows. Two words are related if and only if they share a common letter in their spelling. Is this an equivalence relation?
4. Choose one of the following:
 - (a) Let D_1 and D_2 be subsets of B . Prove $f^{-1}(D_1 \cup D_2) = f^{-1}(D_1) \cup f^{-1}(D_2)$.
 - (b) Let C_1 and C_2 be subsets of A . Prove $f(C_1 \cup C_2) = f(C_1) \cup f(C_2)$.
5. Let $A \subset \mathbb{R}$ be a non-empty subset of the real numbers.
 - (a) What does it mean to say A is bounded?
 - (b) What does it mean to say $b \in \mathbb{R}$ is a lower bound for A ?
 - (c) What does it mean to say $c \in \mathbb{R}$ is the infimum for A ?
6. Complete the following sentence: "Every non-empty subset of \mathbb{R} which is bounded above has a _____."
7. Let $f : A \rightarrow B$ be a function between sets. What is the definition of a left inverse for f ? What is the definition of a right inverse for f ?