Homework 1, Number Systems

Due September 8th, 2007

You can reference anything we did in class. However, if you use something we didn't do in class, then you need to justify it.

1. Prove the following: For all integers m and n,
   (a) $-(m+n) = (-m) + (-n)$.
   (b) $(-m)n = m(-n) = -(mn)$.
2. Prove the following: For all integers m, n, p and q,
   (a) $-(m-n) = n - m$.
   (b) $(m-n) - (p-q) = (m+q) - (n+p)$.
   (c) $(m-n)(p-q) = (mp+nq) - (mq+np)$.
   (d) $m - n = p - q$ if and only if $m + q = n + p$.
3. Rewrite the following statement using the mathematical notation that we discussed in class:

   If the integer x has the property that for some integer m, $m + x = m$, then $x = 0$.

4. Negate the following statements
   (a) Any cubic polynomial has a real root.
   (b) That production of *The Taming of the Shrew* was neither professionally done nor enjoyable.
   (c) A subset of the real numbers is compact if it is closed and bounded.
   (d) H/N is a normal subgroup of G/N if and only if H is a normal subgroup of G.
5. Give some reasoning for why the statement $(P \Rightarrow Q)$ is equivalant to (not $Q \Rightarrow$ not P). Give some examples to help illustrate your point. Try to make your argument one that a non-mathematician can follow.