

Homework 2, Number Systems

Due September 16th, 2007

You can reference anything we did in class. However, if you use something we didn't do in class, then you need to justify it.

1. Prove the following: For all natural numbers n , $n^3 + 5n$ is divisible by 6.
2. Prove the following: For all $n \in \mathbb{N}$, $\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$.
3. Prove the following: If $m \in \mathbb{N}$ and $n \in \mathbb{N}$, then $mn \in \mathbb{N}$.
4. Prove the following: If $m \in \mathbb{Z}$ and $m \neq 0$, then $m^2 \in \mathbb{N}$.
5. Prove the following: The equation $x^2 = -1$ has no solution in \mathbb{Z} .
6. Prove the following: For any $m, n \in \mathbb{Z}$, exactly one of the following is true: $m < n$, $m > n$, or $m = n$.