

Homework 3, Number Systems

Due September 23th, 2007

You can reference anything we did in class. However, if you use something we didn't do in class, then you need to justify it.

1. Write down in set notation as simply as possible the union and intersection of the following two sets A and B , where

$$A = \{2x + 5 \mid x \geq 7, x \in \mathbb{Z}\}$$

and

$$B = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 5\}.$$

What is the complement of A in \mathbb{Z} ? What is the complement of B in \mathbb{Z} ?

2. Determine if the following is true, and if it is true prove it. If it is not true, explain why not.

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C).$$

3. Prove the following: For three sets A , B , and C ,

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

4. Prove the following: If $x \in \mathbb{Z}$, $x < 0$ if and only if $x^3 < 0$.
5. Prove the following: For any $n \in \mathbb{N}$, $5^{2n} - 1$ is divisible by 24.
6. Prove the following: If $m, n \in \mathbb{N}$ and n is divisible by m , then $m \leq n$.
7. Prove the following: Let $n \in \mathbb{Z}$. There is no integer x such that $n < x < n + 1$.