Homework 4, Number Systems

Due October 21st, 2007

You can reference anything we did in class. However, if you use something we didn't do in class, then you need to justify it.

1. Fix $n \in \mathbb{N} - \{1\}$. Prove the following: If $x \equiv a \mod n$ and $y \equiv b \mod n$, then $xy \equiv ab \mod n$.
2. Let Π be a partition of a set A. Define $\sim$ by $a \sim b$ if and only if a and b lie in the same element of Π. Prove that $\sim$ is an equivalence relation on A.
3. Let $f(n)$ denote the sequence of Fibonacci numbers. Prove that for all $m, n \in \mathbb{N}$ where $m \geq 2$, $f(m+n) = f(m-1)f(n) + f(m)f(n+1)$. [Hint: Given $m \geq 2$ prove the following statement using strong induction "For any $n \in \mathbb{N}$, $f(m+n) = f(m-1)f(n) + f(m)f(n+1)$".]
4. Let $f(n)$ denote the sequence of Fibonacci numbers. Prove that for all $m, n \in \mathbb{N}$, $f(mn)$ is divisible by $f(m)$.[Hint: Use the result from the previous problem.]
5. Define a relation $\sim$ on $\mathbb{Z}$ by $x \sim y$ if and only if x divides y or y divides x. Determine if $\sim$ is an equivalence relation. If it is, describe the equivalence classes.
6. Define a relation $\sim$ on the set of all people in the world by saying $A \sim B$ if A and B share a common ancestor within the last five generations. Determine if $\sim$ is an equivalence relation.