Homework 4, Topology

Due Monday November 17, 2008

Justify your answers.

1. Prove that any continuous surjective function from a compact space to a Hausdorff space is a quotient map.
2. Show that every compact subspace of a metric space is closed and bounded. We saw an example in class of a metric space in which not every closed and bounded subspace is compact.
3. Prove that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}$ for any $n > 1$.
4. Let $\mathbb{R}_{fc}$ denote the real numbers with the finite complement topology. Recall that a set $U \subset \mathbb{R}_{fc}$ is open if and only if $\mathbb{R} - U$ is finite.
   (a) Is $\mathbb{R}_{fc}$ connected?
   (b) Is $\mathbb{R}_{fc}$ path connected?
   (c) Is $\mathbb{R}_{fc}$ compact?
5. Let X and Y be topological spaces, X be path connected, and $f : X \to Y$ be a continuous map. Is $f(X)$ necessarily path connected?
6. Let $p : X \to Y$ be a closed, continuous, surjective function such that $p^{-1}(\{y\})$ is compact for each $y \in Y$. Show that if Y is compact, then X is compact. [Hint: If U is an open set containing $p^{-1}(\{y\})$, there is a neighborhood W of y such that $p^{-1}(W)$ is contained in U.]